

Homework 6

0. Problems from sections 10, 13:

10.4.5, 10.4.15, 13.4.2, 13.4.7.

1. Run the Gale–Shapley algorithm on the following rankings of hospitals A , B , C , and doctors x , y , z , to find a stable matching.

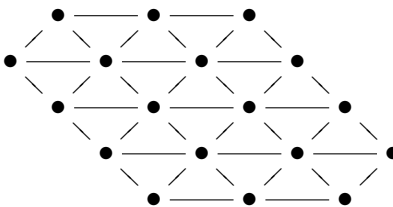
	#1	#2	#3
A	x	y	z
B	x	y	z
C	x	y	z
x	A	B	C
y	A	B	C
z	A	B	C

2. The *girth* of a graph G is the minimum length of a cycle contained as a subgraph of G . Prove that every connected planar graph with n vertices, e edges and girth g satisfies

$$e \leq \frac{g}{g-2}(n-2).$$

3. Prove that the complete bipartite graph $K_{3,3}$ is not planar, but that the complete bipartite graph $K_{2,3}$ is planar. (Do not merely cite Kuratowski's theorem.)

4. What is the chromatic number of the following graph G ?



5. Let T be a tree. Describe an algorithm to produce a 2-coloring of T .

6. Consider the greedy algorithm applied to graph coloring: Starting from a fixed ordering v_1, v_2, \dots, v_n of the vertices of G , consider each vertex in order and color v_i by the smallest positive integer that was not already used to color any neighbor of v_i among v_1, v_2, \dots, v_{i-1} .

Let $\chi(G)$ denote the chromatic number of G . Show that every graph G has a vertex ordering for which the greedy algorithm uses only $\chi(G)$ colors. On the other hand, find a bipartite graph H on $2n$ vertices ordered in such a way that the greedy algorithm uses n colors (rather than $\chi(H) = 2$ colors).