

Homework 8

1. In each part below, find the ordinary generating function of the sequence $\{a_n\}_{n \geq 0}$ (where $a_0 = 1$ for each problem).

(a) a_n is the number of partitions of n with all parts ≤ 4 .

(b) a_n is the number of partitions of n with largest part $= 4$.

(c) a_n is the number of partitions of n with no part appearing more than 2 times.

(d) a_n is the number of partitions of n with no part divisible by 3.

(e) a_n is the number of partitions of n in which each odd part appears at most twice and each even part appears an even number of times.

2. Do problem 4.13.5 from the book.

3. What is the number of essentially different *bracelets* that can be made from 4 beads with k different colors? Here we say that two bracelets are *essentially the same* if one can be rotated *or flipped over* to get the other.

(a) Begin by writing down the automorphism group G of all rotations and flips acting on the vertices of the square. (Hint: All of the rotations are still automorphisms, and we add 4 new flips to get 8 automorphisms in all.)

(b) Next, compute the cycle index Z_G of G .

(c) Apply Pólya's theorem to get a formula for the number of square bracelets that use at most k colors.