Programming Project 1

(1) Write a MATLAB program that uses the power method (as described in 7.3 of our textbook) to compute the dominant eigenvalue λ and a corresponding eigenvector x of any real $n \times n$ matrix A.

INPUT:

- A real $n \times n$ matrix A.
- An initial vector $x_0 \in \mathbb{R}^n$ for the power method. You may use $\begin{bmatrix} 1 \\ 1 \\ \dots \\ 1 \end{bmatrix}$ for x_0 when you run the program. (There is a MATLAB command $ones(10, 1)$ that generates the all 1's vector of size 10.)
- A parameter $\epsilon > 0$ for the convergence check. You may use $\epsilon = 10^{-14}$ when you run the program.

OUTPUT:

- The dominant eigenvalue λ.
- A corresponding eigenvector x.
- The number of iterations performed inside the power method.

For the convergence check, use

$$|\frac{\lambda^{(k)} - \lambda^{(k-1)}}{\lambda^{(k-1)}}| \leq \epsilon$$

(where $\lambda^{(k)}$ is your approximation of the dominant eigenvalue λ after k iterations.)

(2) Test your program for the 10×10 matrix A whose ij entry is $\frac{1}{i+j-1}$, for $1 \leq i, j \leq 10$. (You can generate this in MATLAB using the command $A = hilb(10)$.) Compare the dominant eigenvector that was computed by your program from (1) with the dominant eigenvector of A given by the MATLAB command $[S, D] = eig(A)$. (This command returns a matrix S whose columns are eigenvectors and a diagonal matrix D whose diagonal entries are the corresponding eigenvalues. You can pull off the first column with the command $V(1 : 10, 1)$, for example.)

(3) Suppose you are working for a search engine company. A web-crawling program at your company has produced a 500×500 Markov

matrix A whose entries are the transition probabilities for a subnet of the world-wide web that consists of 500 websites. We call the vector

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_{500} \end{bmatrix} \in \mathbb{R}^{500}$$

the *PageRank* of the subnet if x satisfies

$$Ax = x \qquad \text{and} \qquad \sum_{i=1}^{500} x_i = 1$$

Download the file *A.mat* from the class website, change directory to the place where you downloaded it (for example, use the command *cd~* to change to your home directory), and then run the MATLAB command *load A*; to read in the matrix A. Use your program from (1) to compute the PageRank of the subnet.