Final Exam (Practice)

This is the (practice) final exam for Math 16B, Winter 2009. Please write your name clearly at the top of the exam. You have 120 minutes to complete this exam. You may not use any notes or books, nor any calculating or computing devices. Please give *as much justification as you can* for all of your solutions.

1. Find $y'(x)$ if $x^2 + y^2 = 10$.
2. Find the derivative of $f(x) = \ln(1 - x^2)$.
3. Find $f(x)$ if $f'(x) = \frac{x^2-5}{x^2}$ and $f(1) = 2$.
4. Find the volume of a sphere with radius 2.
5. Find $\int \frac{e^{\sqrt{x+1}}}{\sqrt{x+1}}\,dx$.
6. Find $\int \frac{2x^3}{x^4+1}\,dx$.
7. Find $\int_1^{e^2} x^3 \ln x\,dx$.
8. Find $\int xe^{-2x}\,dx$.
9. Solve the logistic growth equation

$$\frac{dy}{dt} = ky(1 - y)$$

for $y(t)$ where k is a constant, $y > 0$ and $(1 - y) > 0$.

10. Find the indefinite integral $\int \frac{2-x}{(x-1)^3}\,dx$.
11. Find the area bounded by $y = \frac{-4}{x^2+5x+6}$, $x = -1$, $x = 2$ and the x-axis.
12. Evaluate $\int \frac{x^3+3x^2-x-4}{x^2+2x-3}\,dx$.
13. Find $\int_0^\infty \frac{5}{e^{2x}}\,dx$.
14. Find $\int_{-\infty}^{-1} \frac{1}{x^2}\,dx$.
15. Find $\int_{-1/2}^{1/2} \frac{7x+5}{6(2x^2+3x+1)}\,dx$.
16. Given $f(x) = 2x - 2$ on $[1, 2]$, (a) Verify $f(x)$ is a probability density function. (b) Find the mean of the random variable. (c) Find the median of the random varible.
17. Find the variance of the probability density function $f(x) = ce^{-cx}$ on $[0, \infty)$, with $c > 0$.
18. Find the constant k such that $f(x) = k\sqrt{x}(1-x)$ is a probability density function on $[0, 1]$.
19. Find the mean for the probability density function $f(x) = \frac{1}{18}\sqrt{9 - x}$ on $[0, 9]$.