Homework 1

4.1.18. To solve $(x+3)^{4/3} = 16$ for x, raise both sides to the $3/4$ power,

$$((x+3)^{4/3})^{3/4} = 16^{3/4}$$

and simplify to obtain

$$x+3 = (16^{1/4})^3$$
$$x+3 = (2)^3$$

so $x = 8 - 3 = 5$.

4.1.24. The graph of $f(x) = 3^x + 2$ will intersect the y-axis at 3 since $f(0) = 3^0 + 2 = 1 + 2 = 3$. The only graph with this property is graph (b).

4.1.36. (a) If t represents years with $t = 4$ corresponding to 1994, then the year 2006 corresponds to $t = 4 + (2006 - 1994) = 16$. Hence, the sales in 2006 are modeled by

$$S(16) = 116.59(1.3295)^{16} = 11108.99$$

million dollars.

(b) Similarly, the year 2012 corresponds to $t = 4 + (2012 - 1994) = 22$. Hence, the sales in 2012 are modeled by

$$S(22) = 116.59(1.3295)^{22} = 61348.62$$

million dollars.

4.2.12. To solve $x^{-2} = \frac{2}{e^2}$ for x, rewrite this as

$$\frac{1}{x^2} = \frac{2}{e^2}$$

and cross multiply obtaining

$$2x^2 = e^2$$

so

$$x^2 = \frac{e^2}{2}$$

and $x = \pm\frac{e}{\sqrt{2}}$.

4.2.18. The graph of $f(x) = -e^x + 1$ will intersect the y-axis at 0 since $f(0) = -e^0 + 1 = -1 + 1 = 0$. Moreover, the graph will look like the result of reflecting the graph of e^x across the x-axis and then shifting upwards by 1 unit. Graph (a) is the only graph with these properties.

4.2.42. Since the account is compounded monthly, we use the equation

$$A(t) = P(1 + \frac{0.078}{12})^{12t}$$

to determine the account balance after t years. Setting $A(t) = 21154.03$ and $t = 4$, we solve for P to obtain

$$21154.03 = P(1 + \frac{0.078}{12})^{12(4)} = P(1.3648)$$

so $P = 15499.73$ dollars.

4.3.14. To find the derivative of $f(x)$ we apply the chain rule,

$$\frac{d}{dx}\left[\frac{(e^x + e^{-x})^4}{2}\right] = \frac{1}{2}(4)(e^x + e^{-x})^3 \frac{d}{dx}\left[e^x + e^{-x}\right]$$

$$= 2(e^x + e^{-x})^3(\frac{d}{dx}e^x + \frac{d}{dx}e^{-x}) = 2(e^x + e^{-x})^3(e^x - e^{-x})$$

4.3.22. We begin by finding $\frac{dy}{dx}$.

$$\frac{dy}{dx} = 2(e^{4x} - 2)\frac{d}{dx}\left[e^{4x} - 2\right] = 2(e^{4x} - 2)e^{4x}(4) = 8e^{4x}(e^{4x} - 2).$$

Then we evaluate at $x = 0$ to find the slope of the tangent line at the point $(0, 1)$:

$$y'(0) = 8e^{4(0)}(e^{4(0)} - 2) = 8(1)(1 - 2) = -8.$$

Hence, the slope of the tangent line is -8 and it has a y-intercept of 1, because the line passes through the point $(0, 1)$. Therefore, the tangent line has equation

$$y = -8x + 1.$$

4.3.24. To differentiate $x^2y - xe^x + 2 = 0$ implicitly, we apply $\frac{d}{dx}$ to both sides and then isolate $\frac{dy}{dx}$.

$$2xy + x^2\frac{dy}{dx} - (e^x + xe^x) + 0 = 0$$

So,

$$x^2 \frac{dy}{dx} = (e^x + xe^x) - 2xy$$

and

$$\frac{dy}{dx} = \frac{(e^x + xe^x) - 2xy}{x^2}.$$