

## Homework 3

5.1.20. Using the simple power rule, we have

$$\int v^{-1/2} dx = \frac{v^{1/2}}{1/2} + C = 2v^{1/2} + C.$$

Notice that if we differentiate this we get

$$\frac{d}{dv} [2v^{1/2} + C] = \frac{1}{2} 2v^{-1/2} = v^{-1/2}.$$

5.1.28. We rewrite  $\int \frac{1}{8x^3} dx$  as

$$\frac{1}{8} \int x^{-3} dx = \frac{1}{8} \frac{x^{-2}}{-2} + C = -\frac{1}{16} \frac{1}{x^2} + C.$$

5.1.42. We first rewrite this integral as a sum of simpler terms.

$$\int \frac{t^2 + 2}{t^2} dt = \int 1 + \frac{2}{t^2} dt = \int 1 dt + 2 \int t^{-2} dt.$$

Then, using the power rule we have

$$= (t + C_1) + 2\left(\frac{t^{-1}}{-1} + C_2\right) = t - \frac{2}{t} + C.$$

Here, we have combined the constants of integration  $C_1$  and  $C_2$  into a single constant  $C$ . If we differentiate this expression, we recover the integrand.

5.1.60. To find a function  $f(x)$  that satisfies the conditions, we first antidifferentiate  $f''(x) = x^2$ . This gives

$$f'(x) = \frac{x^3}{3} + C$$

for some constant  $C$ . Since  $f'(0) = 6$ , we find that  $C = 6$ . Now, we antidifferentiate  $f'(x) = \frac{x^3}{3} + 6$  to obtain

$$f(x) = \frac{x^4}{12} + 6x + D$$

for some constant  $D$ . Since  $f(0) = 3$ , we find that  $D = 3$ . Hence, we have

$$f(x) = \frac{1}{12}x^4 + 6x + 3.$$

5.2.16. To find the integral, we substitute  $u = 1 - 2x^2$  so  $du = -4x dx$  and  $x dx = -\frac{1}{4} du$ . Using the power rule, we have

$$\int x(1 - 2x^2)^3 dx = -\frac{1}{4} \int u^3 du = -\frac{1}{4} \left( \frac{u^4}{4} + C \right) = -\frac{1}{16} (1 - 2x^2)^4 + C.$$

(As usual, we implicitly redefine the constant of integration  $C$  as necessary when simplifying the answer.) If we differentiate this expression, we recover the integrand.

5.2.26. To find the integral, we substitute  $u = 1 - x^3$  so  $du = -3x^2 dx$  and  $x^2 dx = -\frac{1}{3} du$ . Using the power rule, we have

$$\int \frac{x^2}{\sqrt{1 - x^3}} dx = -\frac{1}{3} \int u^{-1/2} du = -\frac{1}{3} \left( \frac{u^{1/2}}{1/2} + C \right) = -\frac{2}{3} (1 - x^3)^{1/2} + C.$$

(As usual, we implicitly redefine the constant of integration  $C$  as necessary when simplifying the answer.) If we differentiate this expression, we recover the integrand.

5.3.20. To apply the log rule, we first make the substitution  $u = 3 - x^3$  so  $du = -3x^2 dx$  so  $x^2 dx = -\frac{1}{3} du$ . Then,

$$\int \frac{x^2}{3 - x^3} dx = -\frac{1}{3} \int \frac{1}{u} du = -\frac{1}{3} (\ln |u| + C) = -\frac{1}{3} \ln |3 - x^3| + C.$$

5.3.26. To apply the log rule, we first make the substitution  $u = \ln x$  so  $du = \frac{1}{x} dx$ . Then,

$$\int \frac{1}{x(\ln x)^2} dx = \int \frac{1}{u^2} du = \frac{u^{-1}}{-1} + C = -\frac{1}{\ln x} + C.$$

5.3.28. To apply the log rule, we first make the substitution  $u = 1 + e^x$  so  $du = e^x dx$ . Then,

$$\int \frac{e^x}{1 + e^x} dx = \int \frac{1}{u} du = \ln |u| + C = \ln |1 + e^x| + C = \ln(1 + e^x) + C$$

using the fact that  $1 + e^x$  is always positive, so we can drop the absolute value signs.

5.3.48. We rewrite this integral as a sum of simpler terms.

$$\begin{aligned}\int \frac{x-1}{4x} dx &= \int \frac{1}{4} dx - \frac{1}{4} \int \frac{1}{x} dx = \left(\frac{1}{4}x + C_1\right) - \frac{1}{4}(\ln|x| + C_2) \\ &= \frac{1}{4}(x - \ln|x|) + C.\end{aligned}$$

5.3.52. To apply the log rule, we first make the substitution  $u = x + 3$  so  $du = dx$  and  $(x - 3) dx = (u - 6) du$ . Then we have

$$\int \frac{x-3}{x+3} dx = \int \frac{u-6}{u} du.$$

We can rewrite this integral as a sum of simpler terms.

$$\int \frac{u-6}{u} du = \int 1 du - 6 \int \frac{1}{u} du = (u+C) - 6(\ln|u|+C) = (x+3) - 6 \ln|x+3| + C.$$

We could even absorb the 3 into the constant of integration to obtain

$$x - 6 \ln|x+3| + C.$$