# Homework 7

6.6.8. The integrand is continuous on the interval $(\frac{1}{2}, \infty)$ but it diverges to infinity at $x = \frac{1}{2}$. Hence, we must break the integral into two pieces, say at $x = 1$. By definition

$$\int_{1/2}^{\infty} \frac{1}{\sqrt{2x-1}}\,dx = \int_{1/2}^{1} \frac{1}{\sqrt{2x-1}} + \int_{1}^{\infty} \frac{1}{\sqrt{2x-1}}\,dx$$

$$= \lim_{a \to (1/2)^+} \int_a^1 (2x-1)^{-1/2}\,dx + \lim_{b \to \infty} \int_1^b (2x-1)^{-1/2}\,dx.$$

The improper integral diverges if either of these limits do not exist. We evaluate the second limit using the substitution $u = 2x - 1$ so $du = 2\,dx$ and the limits of integration become 1 and $2b-1$, respectively. Then, the second limit is

$$= \lim_{b \to \infty} \frac{1}{2} \int_1^{2b-1} u^{-1/2}\,du = \lim_{b \to \infty} \frac{1}{2} \left[2u^{1/2}\right]_1^{2b-1}$$

$$= \lim_{b \to \infty} \frac{1}{2} \left[2(2b-1)^{1/2} - 2\right] = \lim_{b \to \infty} (2b-1)^{1/2} - 1 = \infty$$

so the improper integral diverges.

6.6.12. The integrand is continuous on the interval $(-\infty, 0]$. By definition,

$$\int_{-\infty}^{0} \frac{x}{x^2+1}\,dx = \lim_{a \to -\infty} \int_a^0 \frac{x}{x^2+1}\,dx$$

and using the substitution $u = x^2 + 1$ so $du = 2x\,dx$, this becomes

$$= \lim_{a \to -\infty} \frac{1}{2} \int_{a^2+1}^{1} \frac{1}{u}\,du = \lim_{a \to -\infty} \frac{1}{2} \left[\ln|u|\right]_{a^2+1}^{1}$$

$$= \lim_{a \to -\infty} \frac{1}{2}(0 - \ln(a^2+1)) = \lim_{a \to -\infty} -\frac{1}{2} \ln(a^2+1) = -\infty$$

so the improper integral diverges.

6.6.18. The integrand is continuous on the interval $[0, 2]$ except at $x = 1$ where it tends to infinity. Hence, we must break the integral into two pieces at $x = 1$. By definition

$$\int_0^2 \frac{1}{(x-1)^2}\,dx = \int_0^1 \frac{1}{(x-1)^2}\,dx + \int_1^2 \frac{1}{(x-1)^2}\,dx$$

$$= \lim_{b\to 1^-} \int_0^b (x-1)^{-2}\,dx + \lim_{a\to 1^+} \int_a^2 (x-1)^{-2}\,dx$$

$$= \lim_{b\to 1^-} \left[-(x-1)^{-1}\right]_0^b + \lim_{a\to 1^+} \left[-(x-1)^{-1}\right]_a^2$$

$$= \lim_{b\to 1^-} \left[-\frac{1}{b-1} - 1\right] + \lim_{a\to 1^+} \left[-1 + \frac{1}{a-1}\right].$$

Since these limits diverge, the improper integral diverges.

6.6.24. The integrand is continuous on the interval $(0,1]$ so by definition

$$\int_0^1 \frac{1}{x}\,dx = \lim_{a\to 0^+} \int_a^1 \frac{1}{x}\,dx = \lim_{a\to 0^+} \left[\ln|x|\right]_a^1$$

$$= \lim_{a\to 0^+} (0 - \ln a) = \infty.$$

so the improper integral diverges.

9.1.4. (a) The sample space $S$ is set of all possible outcomes of the experiment. In this case, $S$ consists of the $3^3 = 27$ possible ordered lists of $I$'s, $O$'s, and $U$'s. Explicitly,

$$S = \{III, IIO, IIU, IOI, IOO, IOU, IUI, IUO, IUU,$$

$$OII, OIO, OIU, OOI, OOO, OOU, OUI, OUO, OUU,$$

$$UII, UIO, UIU, UOI, UOO, UOU, UUI, UUO, UUU\}.$$

(b) The event that at least two people are in favor consists of the subset of $S$ with two or three $I$'s. This is

$$A = \{III, IIO, IIU, IOI, IUI, OII, UII\}.$$

(c) The event that no more than one person is opposed consists of the subset of $S$ with zero or one $O$'s. This is

$$B = \{III, IIO, IIU, IOI, IOU, IUI, IUO, IUU,$$

$$OII, OIU, OUI, OUU, UII, UIO, UIU, UOI, UOU, UUI, UUO, UUU\}.$$

9.1.22. We have

$$E(x) = 1\frac{4}{10} + 2\frac{2}{10} + 3\frac{2}{10} + 4\frac{1}{10} + 5\frac{1}{10} = \frac{23}{10} = 2.3$$

and

$$V(x) = \left(1 - \frac{23}{10}\right)^2 \frac{4}{10} + \left(2 - \frac{23}{10}\right)^2 \frac{2}{10} + \left(3 - \frac{23}{10}\right)^2 \frac{2}{10} + \left(4 - \frac{23}{10}\right)^2 \frac{1}{10} + \left(5 - \frac{23}{10}\right)^2 \frac{1}{10}$$

$$= \frac{181}{100} = 1.81$$

and

$$\sigma = \sqrt{\frac{181}{100}} = 1.35.$$