Assignment 5

(due date: Wednesday, Nov 5)

For this assigment use the general definitions of convergence, continuity, and compactness, in an arbitrary topological space. X and Y denote topological spaces.

Problem 1. Prove that $K \subset X$ is a compact subset of X in the topology $\mathcal{T}$ iff $(K, \mathcal{T}_K)$ is a compact topological space (i.e., is a compact subset of itself), for the relative topology $\mathcal{T}_K$ on K, induced by $\mathcal{T}$.

Problem 2. Consider $f : X \to Y$. Prove that f is continuous iff for every open $G \subset Y$, $f^{-1}(G)$ is open in X.

Problem 3. Let $f : X \to Y$ be continuous and let (x_n) be a sequence in X converging to $x \in X$. Prove that $(f(x_n))$ converges to $f(x) \in Y$.

Problem 4. Let $f : X \to Y$ be continuous. Prove that for every compact $K \subset X$, $f(K)$ is a compact subset of Y.

Problem 5. Let K be a compact topological space. Prove that any closed subset $A \subset K$ is compact.

Problem 6. Let (x_n) be a sequence in a Hausdorff space X, and $x, y \in X$ such that $x_n \to x$ and $x_n \to y$. Prove that $x = y$.

Problem 7. Let X be a Hausdorff space and $K \subset X$ compact. Prove that K is closed.