Assignment 3

(due date: Monday, January 26)

Problem 1. Find the Fourier series for the following two functions in $L^2([0, 2\pi])$:

a)

$$f_1(x) = \begin{cases} \pi^{-1}x & \text{if } 0 \le x \le \pi \\ 2 - \pi^{-1}x & \text{if } \pi \le x \le 2\pi \end{cases}$$

b)

$$f_2(x) = \begin{cases} (2\pi)^{-1}x & \text{if } 0 \le x < 2\pi \\ 0 & \text{if } x = 2\pi \end{cases}$$

Problem 2. Prove that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

(Hint: use the solution of Problem 1 b) and Parseval's identity.)

Problem 3. Prove Wirtinger's inequality which states that for $f \in C^1([a, b)$, with $f(a) = f(b) = 0$,

$$\int_a^b |f(x)|^2 \, dx \le \frac{(b-a)^2}{\pi^2} \int_a^b |f'(x)|^2 \, dx$$

(Hint: First, consider $a = 0, b = \pi$ and apply Parseval's identity to the odd extension of f' to the interval $[-\pi, \pi]$. Then use the relation between the Fourier cooeficients of f and f'.

Problem 4. Exercise 10.1 of the notes.