Assignment 6

(due date: Monday, February 23)

Problem 1. Suppose $A, B \in \mathcal{B}(\mathcal{H})$ satisfy

$$\langle x, Ay\rangle = \langle x, By\rangle, \quad \text{for all } x, y \in \mathcal{H} \quad .$$

Prove that it follows that $A = B$.

Problem 2. Prove the following properties of the adjoint: for all $A, B \in \mathcal{B}(\mathcal{H})$,and $\lambda \in \mathbb{C}$ one has

a) $A^{**} = A$.

b) $(AB)^* = B^*A^*$.

c) $(\lambda A)^* = \overline{\lambda} A^*$.

d) $(A + B)^* = A^* + B^*$.

e) $\|A^*\| = \|A\|$.

f) $\|A^*A\| = \|A\|^2$.

Problem 3. Show by an example that, in general, $\|AA\| \neq \|A\|^2$.

Problem 4. Let $\{u_n\}$ be a sequence of orthonormal vectors in a Hilbert space. Prove that $u_n \to 0$ weakly.