Homework #4. Due: Tuesday, 14 March 2006

Both problems of this homework are about proving absence of symmetry breaking in one-dimensional quantum spin systems. Both can be done as an application of the general energy-entropy balance result discussed in class.

Problem 1. Consider a one-dimensional quantum spin model with n-dimensional single-site Hilbert spaces and an interaction Φ and a local symmetry, τ, defined by a unitary $U \in M_n(\mathbb{C})$:

$$\tau(A) = \left(\bigotimes_{x \in \Lambda} U^* \right) A \left(\bigotimes_{x \in \Lambda} U \right), \quad \text{for all } A \in \mathcal{A}_\Lambda \,.$$

Assume that Φ is such that

$$\sup_{x \in \mathbb{Z}} \sum_{X \subset, X \ni x} \operatorname{diam}(X) \|\Phi\| < +\infty$$

where $\operatorname{diam}(X)$ is defined as the length of the shortest interval containing X. Prove that all $\beta-$KMS states for the corresponding model, for all $\beta \in [0, \infty)$, are $\tau-$invariant.

Problem 2. Consider a one-dimensional quantum spin model with n-dimensional single-site Hilbert spaces and a translation invariant interaction of the following form:

$$H_\Lambda = \sum_{x,y \in \Lambda} J(x-y) \Phi_{x,y} + \sum_{x \in \Lambda} \Phi_x \,,$$

where $\Phi_{x,y} \in \mathcal{A}_{\{x,y\}}$ and $\Phi_x \in \mathcal{A}_{\{x\}}$ satisfy $\Phi_{x+a,y+a} = \Phi_{x,y}$ and $\Phi_{x+a} = \Phi_x$, and $\|\Phi_{x,y}\| \leq 1$, for all $a, x, y \in \mathbb{Z}$. Let τ denote the automorphism of translation to the right by one lattice unit. Assume the following two conditions on the coupling constants $J(x)$:

$$\sum_{x \in \mathbb{Z}} |J(x)| < +\infty \tag{1}$$

and

$$\sum_{x \in \mathbb{Z}} |x| |J(x+1) - J(x)| < +\infty \tag{2}$$

a) Prove that all $\beta-$KMS states are $\tau-$invariant.

b) Prove that if $J(x) = J(|x|) \geq 0$ and monotone decreasing in $|x|$, then (1) implies (2).

c) Give an example of coupling constants $J(x)$ such that (1) holds but not (2).