MAT 21C: PRACTICE PROBLEMS LECTURE 1

PROFESSOR CASALS (SECTIONS B01-08)

Abstract. These practice problems correspond to the first lecture of Part II, delivered on May 1st 2023.

Problem 1. Consider the line $L := \{3x - 7y = 4\}$, i.e. points (x, y) in the plane that satisfy the equation $3x - 7y = 4$.

(1) Decide which of the following points belongs to L:

$$(0,0), \quad (1,0), \quad (-1,-1), \quad (-2,3), \quad (6,2), \quad (-8,-6).$$

(2) Draw the line L in the plane.

For (1), we substitute the values of x and y for every point into the equation $3x-7y = 4$ and see if it is satisfied:

- For $(0,0)$: $3 \cdot 0 - 7 \cdot 0 \neq 4$, and thus $(0,0)$ does *not* belong to L.
- For $(1,0)$: $3 \cdot 1 - 7 \cdot 0 \neq 4$, and thus $(1,0)$ does *not* belong to L.
- For $(-1,-1)$: $3 \cdot (-1) - 7 \cdot (-1) = 4$, and thus $(-1,-1)$ does belong to L.
- For $(-2,3)$: $3 \cdot (-2) - 7 \cdot 3 \neq 4$, and thus $(-2,3)$ does *not* belong to L.
- For $(6,2)$: $3 \cdot 6 - 7 \cdot 2 = 4$, and thus $(6,2)$ does belong to L.
- For $(-8,-4)$: $3 \cdot (-8) - 7 \cdot (-4) = 4$, and thus $(-8,-6)$ does belong to L.

So the points that belong to L are $(-1,-1), (6,2)$ and $(-8,-6)$, while the rest do not.

For (2), plot L the unique *straight* line that passes through any of two of the points above. For instance, L is the unique line through $(-1,-1)$ and $(6,2)$.

Problem 2. Consider the line $L = \{2x - y = 3\}$ in the plane.

(i) Find two different points that belong to L.
(ii) Find two different points that do *not* belong to L

For (i), by writing the equation of L as $y = 2x - 3$ we can give values to x and solve for y. For instance, if we set $x = 0$ we obtain $y = -3$. Thus, the point $(0,-3)$ is in L. If we set $x = 1$ we obtain $y = -1$. Thus, the point $(1,-1)$ is also in L. Two different points that belong to L are thus $(0,-3)$ and $(1,-1)$.

For (ii), the origin $(0,0)$ and $(1,0)$ are to instances of points that do not belong to L.

Problem 3. Consider the set $H = \{2x - y \geq 3\}$ in the plane.

(i) Explain why the line $L = \{2x - y = 3\}$ is contained in the set H.
(ii) Find a point in H which is not in L.
(iii) Draw the set H in the plane.

For (i), all points (x, y) that belong to L must solve the equation $2x - y = 3$. In particular, they satisfy $2x - y \geq 3$ and thus also belong to H.

For (ii), the point $(4, 0)$ satisfies $2 \cdot 4 - y \cdot 0 \geq 3$ but does not satisfy $2 \cdot 4 - y \cdot 0 = 3$. Therefore $(4, 0)$ belongs to H but not L.

For (iii), the line L divides the plane into two halves. The set H is exactly the half which lies below the line L. We know it must be the lower half because $(4, 0)$ belongs to H and $(4, 0)$ lies beneath L.

Problem 4. Consider the circle $C = \{x^2 + y^2 = 9\}$.

(a) Verify that all four points $(3, 0)$, $(-3, 0)$, $(0, -3)$ and $(0, 3)$ belong to C.

(b) Find two points different from the ones in Part (a) that also belong to C.

(c) Draw the circle C in the plane.

(d) Can you describe the region of points that satisfy $D = \{x^2 + y^2 \leq 9\}$.

For (a), note that $(\pm 3)^2 + 0^2 = 9$, and thus $(\pm 3, 0)$ belongs to C. Similary, $0^2 + (\pm 3)^2 = 9$ and $(0, \pm 3)$ belongs to C as well.

For (b), the point $(\sqrt{3}/2, \sqrt{3}/2)$, or $(-\sqrt{3}/2, \sqrt{3}/2)$ belong to C. Similary, both $(\sqrt{3}/2, \pm\sqrt{3}/2)$ belong to C. These points can be found by re-writing the equation of C as

$$y = \pm\sqrt{9 - x^2}$$

and plugging a value for x. In the last two examples we chose $x = \sqrt{3}/2$. so that $y = \pm\sqrt{3}/2$.

For (c), you should draw C as the unique circle centered at the origin with radius 3. The equation is indeed the equation of a circle $x^2 + y^2 = r^2$ in the plane with radius $r = 3$

For (d), we note that the points of the circle C also satisfy $\{x^2 + y^2 \leq 9\}$. Hence, C is contained in D. Furthermore, the origin $(0, 0) \in D$ is contained in D because $0^2 + 0^2 \leq 9$. Note that the circle C divides the plane into two pieces: its inside and the outside. H is the unique piece that contains $(0, 0)$. Hence, H must be the disk (of radius 3) bounded by the circle C.