MAT 21C: PRACTICE PROBLEMS LECTURE 4

PROFESSOR CASALS (SECTIONS B01-08)

ABSTRACT. Practice problems for the fourth lecture of Part II, delivered May 8 2023. Solutions will be posted within 48h of these problems being posted.

Problem 1. Consider the vectors $v = \langle 3, -7, 1\rangle$ and $w = \langle -1, 4, 2\rangle$.

(a) Compute the following vectors $v + w$, $2v$, $5w$ and $2v + 5w$.
(b) Find the length of v and w.
(c) Find the unit length vector in the direction of v and the unit length vector in the direction of w.

Problem 2. Consider the points $P = (-5, 1, 2)$, $Q = (7, 6, -3)$, and $R = (1, 0, 4)$.

(a) Find the vector $\vec{PQ}$ and its length $|\vec{PQ}|$.
(b) Find the midpoint between P and Q.
(c) Compute the vectors $\vec{QR}$ and $\vec{PR}$ and show that $\vec{PQ} + \vec{QR} = \vec{PR}$.

Problem 3. Consider the vectors $v = \langle 1, -2, -3\rangle$ and $w = \langle -1, 4, 5\rangle$.

(a) Find a vector u such that $v + u = w$.
(b) Compute the lengths $|v|, |w|$ and $|v + w|$.
(c) Justify geometrically that $|v + w| \leq |v| + |w|$.

Problem 4. Suppose that a vector $v = \langle v_1, v_2, v_3\rangle$ has length zero $|v| = 0$. Argue that then $v = \langle 0, 0, 0\rangle$, i.e. $v_1 = v_2 = v_3 = 0$.

Problem 5. (Optional) Consider the eight vectors

$$v_1 = \langle 1, 1, 1\rangle, \quad v_2 = \langle 1, -1, 1\rangle, \quad v_3 = \langle 1, 1, -1\rangle, \quad v_4 = \langle -1, -1, 1\rangle$$
$$v_5 = \langle -1, 1, 1\rangle, \quad v_6 = \langle 1, -1, -1\rangle, \quad v_7 = \langle -1, 1, -1\rangle, \quad v_8 = \langle -1, -1, -1\rangle,$$

which we think of geometrically as starting at the origin $O = (0, 0, 0)$.

(a) Draw the endpoints of each of the 8 vectors v_i, $i = 1, \ldots, 8$.

(b) Show by direct computation that
$$v_1 + v_2 + v_3 + v_4 + v_5 + v_6 + v_7 + v_8 = \vec{0},$$
where $\vec{0} = (0, 0, 0)$ is the zero vector.

(c) Justify geometrically that
$$v_1 + v_2 + v_3 + v_4 + v_5 + v_6 + v_7 + v_8 = \vec{0},$$
by thinking about these vectors as arrows going around a cube.