MAT 21C: PRACTICE PROBLEMS LECTURE 6

PROFESSOR CASALS (SECTIONS B01-08)

Abstract. Practice problems for the sixth lecture of Part II, delivered May 12 2023. Solutions will be posted within 48h of these problems being posted.

Problem 1. Consider the vectors $v = \langle -2, 1, -1\rangle$ and $u = \langle 4, -5, 7\rangle$.

(a) Compute the cross product $u \times v$.
(b) By explicit computation and only for this example, verify that the cross product $u \times v$ is orthogonal to both u and v.
(c) Find the area of the parallelogram spanned by u and v.

Problem 2. Consider the vectors $v = \langle 3, -7, 1\rangle$ and $w = \langle -1, 4, 2\rangle$.

(a) Show that the cross product is $u \times v = \langle -18, -7, 5\rangle$.
(b) Find an equation for the plane π which contains both u and v and passes through the origin $(0, 0, 0)$.

Problem 3. Consider the parallelogram with vertices $(0, 0, 0), (4, 5, -11), (-3, 2, 17), (1, 7, 6)$.

(a) Show that the parallelogram is spanned by $u = \langle 4, 5, -11\rangle$ and $v = \langle -3, 2, 17\rangle$.
(b) Find the area of the parallelogram.

Problem 4. Let u, v be two vectors.

(a) Show by direct computation that $u \times v = -v \times u$.
(b) Argue geometrically that $u \times v = -v \times u$.

Problem 5. Suppose that $u = \langle u_1, u_2, u_3\rangle, v = \langle v_1, v_2, v_3\rangle$ are vectors.

(a) Prove by direct computation that u, v are parallel, i.e. we have equality of the ratios $\frac{u_1}{v_1} = \frac{u_2}{v_2} = \frac{u_3}{v_3}$, if and only if $u \times v$=0.
(b) Using the cross product angle formula show that two vectors u, v parallel if and only if $u \times v = 0$.
(c) Justify geometrically that u, v parallel if and only if $u \times v = 0$.

Problem 6. Using the cross product, find an equation for the unique plane π containing the points $(0, 0, 0)$, $(2, -5, -8)$ and $(11, -7, 34)$.