MAT 21C: PRACTICE PROBLEMS LECTURE 7

PROFESSOR CASALS (SECTIONS B01-08)

ABSTRACT. Practice problems for the seventh lecture of Part II, delivered May 15 2023. Solutions will be posted within 48h of these problems being posted.

Recall the four descriptions of a plane π:

(1) The plane π is given by three points $P, Q, R \in \pi$.
(2) The plane π is given by one point $P \in \pi$ and a normal direction $n = \langle a, b, c \rangle$.
(3) The plane π is given by one point $P \in \pi$ and two vectors u, v inside of π.[1]
(4) The plane π is given by an equation

$$\pi = \{ax + by + cz = d\},$$

where $a, b, c, d \in \mathbb{R}$ are real numbers.

Problem 1. Consider the unique plane π containing the three points $P = (1, 1, 2), Q = (-2, 3, 0)$ and $R = (0, -5, 7)$.

(a) Find two vectors u, v inside of π.
(b) Compute a perpendicular direction to π.
(c) Find an equation for π.

Problem 2. Find an equation for the plane through point $(9, 3, -1)$ parallel to the plane $\{x + y + z = 0\}$.

Problem 3. Consider the three planes

$$\pi_1 = \{3x - 5y + 4z = 12\}$$
$$\pi_2 = \{\text{unique plane that contains } (0, 1, 0) \text{ with perpendicular direction } \langle 1, 4, 3 \rangle\}$$
$$\pi_3 = \{\text{unique plane that contains } (0, 0, 0) \text{ and vectors } u = \langle 2, 4, 1 \rangle, v = \langle 2, -5, 12 \rangle\}$$

(a) Show that π_1 intersects π_2 at a line, π_1 intersects π_3 at a line, and π_2 intersects π_3 at a line. (That is, these are not parallel to each other.)

(b) Find the directions of each of these lines.

[1] It is fine if u, v are just two vectors in the direction parallel to π.

Problem 4. Consider the two planes

$$\pi_1 = \{3x + 3y + 3z = 12\}$$
$$\pi_2 = \{\text{unique plane that contains } (0,0,0) \text{ with perpendicular direction } \langle 1,1,1\rangle\}$$

(a) Show that π_1 and π_2 are parallel planes and they are different.

(b) Find a plane π_3 different than π_1 and π_2 but is parallel to both of them.

Problem 5. Consider the plane $\pi = \{2x + 9y - z = 3\}$.

(a) Find three distinct points $P, Q, R \in \pi$ that belong to π.

(b) Find two vectors u, v which are parallel to π.

(c) Find a plane π' parallel to π but different from it.

(d) Find a plane π'' which intersects π at a line.

Problem 6. Consider the plane $\pi = \{2x + y - z = 0\}$ and the unique line L through the origin and the point $P = (0, 1, 1)$.

(a) Argue that the point $P \in \pi$ belongs to the plane π.

(b) Justify that the line L lies inside the plane π.

(c) Find a plane π' such that their intersection is the line L.