University of California Davis **Name (Print):** ____________
Calculus MAT 21C **Student ID (Print):** ____________

Midterm Examination **April 28 2023**
Time Limit: 50 Minutes

This examination document contains 8 pages, including this cover page, and 4 problems. You must verify whether there any pages missing, in which case you should let the instructor know. **Fill in** all the requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

The maximum grade for this exam is **100 points**. Any grade surpassing that mark will still count for 100 points, which is the top grade. You can obtain 100 points in any combination of the problems below, the **extra 20 bonus points** can only help you. You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

(A) **If you use a lemma, proposition or theorem which we have seen in the class or in the book, you must indicate this** and explain why the theorem may be applied.

(B) **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive little credit.

(C) **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by no calculations, explanations, or algebraic work will receive less credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.

(D) If you need more space, use back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	30	
2	40	
3	30	
4	20	
Total:	120	

Do not write in the table to the right.

1. (30 points) Consider the sequence $a_n = \dfrac{2^n}{n!}$, where n denotes a natural number $n \geq 1$.

   (a) (5 points) Write the first 4 terms a_1, a_2, a_3, a_4 of this sequence.

   (b) (5 points) Justify that $a_{n+1} \leq a_n$ for all n. (So the sequence (a_n) is decreasing.)

(c) (5 points) Argue that $a_n \geq 0$ for all $n \geq 1$. (So the sequence (a_n) is bounded below.)

(d) (10 points) Show that (a_n) is convergent.

(e) (5 points) Explain why the limit $\lim_{n\to\infty} a_n = 0$ is 0.

2. (40 points) Solve the two parts below

   (a) (36 points) For each of the series below, determine whether the series converges or diverges. Each answer is worth 9 points. You *must* justify your answer in detail. If you are applying a certain test, **state the name of the test clearly, the steps implementing the test and its outcome**. If a sequence converges, you do *not* need to find the limit.

$$1. \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n+1}}, \qquad 2. \sum_{n=1}^{\infty} \frac{2n^6 + 15n^2 + 1}{n^6 - 9n^2 - 10}, \qquad 3. \sum_{n=1}^{\infty} \frac{3^n}{n!}, \qquad 4. \sum_{n=1}^{\infty} \frac{\ln(n)^n}{n^n}.$$

(b) (4 points) Explain why the following series converges if and only if the positive real value α satisfies $\alpha > 2$:

$$\sum_{n=1}^{\infty} \frac{n}{n^\alpha + 14}.$$

3. (30 points) Consider the function $f(x) = \cos(x)$.

   (a) (10 points) Compute the five values $f(0)$, $f'(0)$ and $f''(0)$, $f'''(0)$ and $f''''(0)$, i.e. of $\cos(0)$ and the first fourth derivatives of $\cos(x)$ at 0.

   (b) (10 points) Argue that the Taylor expansion of $\cos(x)$ of order 4 at $a = 0$ is

$$\cos(x) \approx 1 - \frac{x^2}{2} + \frac{x^4}{4!}.$$

(c) (10 points) Explain why the Taylor expansion of $\cos(x)$ of order 4 at $a = 0$ approximates the value $\cos(0.1)$ at $x = 0.1$ with an error less than 10^{-5}.

Hint: We saw in lecture that the error $R_4(x)$ of the Taylor expansion of $\cos(x)$ *of order 4 centered at $a = 0$ is bounded above by $\frac{1}{5!}x^5$. You can use that.*

4. (20 points) For each of the five sentences below, circle the correct answer. There is a unique correct answer per item. (You do *not* need to justify your answer.)

   (a) (4 points) The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ for p positive real number

   (1) converges if $p \geq 1$. (2) converges if $p > 1$. (3) converges for all p.

   (b) (4 points) The value of the infinite series $\sum_{n=0}^{\infty} \frac{1}{6^n}$ is

   (1) 1/6 (2) 6/5 (3) ∞

   (c) (4 points) The radius of convergence of the power series $\sum_{n=0}^{\infty} x^n$ is

   (1) 0 (2) 1/2 (3) 1

   (d) (4 points) The Taylor series of the polynomial $f(x) = 1 - 3x + x^2$ at $a = 0$ is

   (1) $1 - 3x + x^2$ (2) $1 - 3x + x^2 + x^3 + x^4$ (3) $1 - 3x + x^2 - 3x^3 + x^4$

   (e) (4 points) The Taylor expansion of $x^2 e^{-x}$ at $x = 0$ of order 3 (so the trucantion with 4 terms: those with a constant, x, x^2 and x^3) is

   (1) $1 - x + \frac{x^2}{2!} - \frac{x^3}{3!}$ (2) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$ (3) $x^2 - x^3$