University of California Davis **Name (Print):** ____________________
Calculus MAT 21C **Student ID (Print):** ____________________

Practice Midterm Examination **April 28 2023**
Time Limit: 50 Minutes

This examination document contains 8 pages, including this cover page, and 4 problems. You must verify whether there any pages missing, in which case you should let the instructor know. **Fill in** all the requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

(A) **If you use a lemma, proposition or theorem which we have seen in the class or in the book, you must indicate this** and explain why the theorem may be applied.

(B) **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive little credit.

(C) **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive little credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.

(D) If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	25	
2	25	
3	25	
4	25	
Total:	100	

1. (25 points) Consider the sequence (a_n) defined recursively by $a_{n+1} = \sqrt[3]{a_n^2 + a_n + 2}$ and initial condition $a_1 = 0$.

   (a) (15 points) Show that the sequence (a_n) is convergent.

(b) (10 points) Find the limit of the sequence (a_n).

2. (25 points) Solve the two parts below

   (a) (20 points) For each of the series below, determine whether the series converges or diverges. You *must* justify your answer in detail. If you are applying a certain test, state the name of the test clearly, the steps implementing the test and its outcome. If a sequence converges, you do *not* need to find the limit.

$$1. \sum_{n=1}^{\infty} \frac{\cos(\ln(n))}{12^n}, \qquad 2. \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^5+3n+6}}, \qquad 3. \sum_{n=1}^{\infty} \frac{\ln(n)}{n^n}, \qquad 4. \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}.$$

(b) (5 points) Discuss for which positive real values of $\alpha \in (0, \infty)$ this series converges:

$$\sum_{n=1}^{\infty} \frac{n^3}{1+n^\alpha}.$$

3. (25 points) Solve the following parts.

   (a) (6 points) Find the Taylor expansion of $x^2 \cos(x)$ of order 8 at $x = 0$.

   (b) (6 points) Find the Taylor expansion of $f(x) = x^2 \cos(x^3)$ of order 20 at $x = 0$.

(c) (6 points) Compute the radius of convergence of the Taylor series of $f(x)$ at $x = 0$.

(d) (7 points) What is the approximated value of $f(0.1)$ given by the Taylor approximation of order 20 at $x = 0.1$?

4. (25 points) For each of the ten sentences below, circle whether they are **true** or **false**. (You do *not* need to justify your answer.)

   (a) (5 points) If a sequence (a_n) converges to 0, then $\sum_{n=1}^{\infty} a_n$ converges.

   (1) True. (2) False.

   (b) (5 points) If a power series $A(x)$ (centered at $x = 0$) diverges for $x = 12$, then it diverges for $x = -14$.

   (1) True. (2) False.

   (c) (5 points) The Taylor series of a non-constant differentiable function $f : \mathbb{R} \to \mathbb{R}$ at $x = 0$ cannot have all its Taylor coefficients be equal to zero.

   (1) True. (2) False.

   (d) (5 points) The Taylor series associated to a real polynomial centered at *any* point $x = a$, $a \in (-\infty, \infty)$, always converges.

   (1) True. (2) False.

   (e) (5 points) If the ratio test can determine the convergence of a convergent series $\sum_{n=1}^{\infty} a_n$, then so can the root test.

   (1) True. (2) False.