University of California Davis **Name (Print):** ____________________

Calculus MAT 21C **Student ID (Print):** ____________________

Practice Midterm II Examination **May 26 2023**

Time Limit: 50 Minutes

This examination document contains 8 pages, including this cover page, and 4 problems. You must verify whether there any pages missing, in which case you should let the instructor know. **Fill in** all the requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

(A) **If you use a lemma, proposition or theorem which we have seen in the class or in the book, you must indicate this** and explain why the theorem may be applied.

(B) **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive little credit.

(C) **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive little credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.

(D) If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	25	
2	25	
3	25	
4	25	
Total:	100	

1. (25 points) Consider the points $P = (1, 0, 0)$, $Q = (-2, 0, 3)$ and $R = (-5, 1, -1)$.

   (a) (5 points) Compute the vector $\vec{PQ} \times \vec{PR}$.

   (b) (5 points) Consider the unique plane π containing P, Q and R. Explain why

   $$\{-3x - 21y - 3z = -3\}$$

   is an equation for π.

(c) (5 points) Justify that $v = (21, 0, -21)$ is a direction of the line L of intersection of π with the plane $\Pi = \{x + z = 1\}$.

(d) (5 points) Find the distance from $S = (0, 0, 2)$ to line L.

(e) (5 points) Find the distance from $S = (0, 0, 2)$ to the plane π.

2. (25 points) Consider the vectors $u = \langle 2, 0, -1 \rangle$ and $v = \langle 3, 4, -5 \rangle$.

   (a) (5 points) Show that $\langle 4, 7, 8 \rangle$ is perpendicular to both u and v.

   (b) (5 points) Argue that u is not parallel to v.

(c) (5 points) Compute $\sin\theta$, where θ is the angle between u and v.

(d) (5 points) Verify that the vector $w = \langle 1, 0, 2\rangle$ is perpendicular to u but w is *not* perpendicular to v.

(e) (5 points) Find a vector that is perpendicular to v but *not* perpendicular to u.

3. (25 points) Consider a particle moving with a trajectory $\vec{r}(t) = \langle \cos(3t), \sin(4t), t^3 \rangle$.

   (a) (5 points) Where will the particle be at time $t = \pi$?

   (b) (5 points) Find the velocity vector $\vec{v}(t)$ of the particle.

   (c) (5 points) Compute the speed of the particle at time $t = \pi$.

(d) (5 points) Show that the acceleration at $t = \pi$ is given by

$$a(\pi) = \langle 9, 0, 6\pi \rangle.$$

(e) (5 points) Will there ever be a positive time t where the particle will be at rest, i.e. have zero speed?

4. (25 points) For each of the statements below, circle the **unique** correct answer. (You do *not* need to justify your answer.)

   (a) (5 points) The intersection of the ball $(x-2)^2+y^2+(z+1)^2 \leq 16$ with the plane $\pi = \{2x - 11y + 5z = -1\}$ is:

   (1) Empty. (2) A circle. (3) A disk. (4) A half-space. (5) A line.

   (b) (5 points) The intersection of the plane $\pi_1 = \{x + y + z = 1\}$ with the plane $\pi_2 = \{5x + 5y + 5z = 17\}$ is:

   (1) Empty. (2) A circle. (3) A line. (4) A point. (5) Two points.

   (c) (5 points) The cross product of $u = \langle -3, 2, 4\rangle$ and $v = \langle 6, -4, -8\rangle$:

   (1) $\langle 0, 0, 0\rangle$ (2) $\langle 1, 0, 0\rangle$ (3) $\langle 0, 1, 0\rangle$ (4) $\langle 0, 0, 1\rangle$ (5) $\langle 1, 1, 1\rangle$.

   (d) (5 points) The midpoint between $P = (0, 6, 4)$ and $Q = (8, 2, -4)$ is:

   (1) $\langle 4, 4, 0\rangle$ (2) $\langle 4, -2, -4\rangle$ (3) $\langle 8, -4, 8\rangle$ (4) $\langle 0, -8, 4\rangle$ (5) $\langle 2, 0, -4\rangle$.

   (e) (5 points) A particle with trajectory $r(t) = (e^t, t + 3, 5t)$ has speed at $t = 0$:

   (1) 0. (2) $\sqrt{25}$. (3) $\sqrt{26}$. (4) $\sqrt{27}$.