1. Linear algebra:

Goal: to solve linear systems of equations

Example: $x_1, x_2, x_3 \in \mathbb{R}$ → real no.

unknown variables

$$\begin{cases} 3x_1 - 4x_2 + \ln(2)x_3 = -2 & \dots ① \\ x_1 + 7x_2 - e^{37}x_3 = \cos(375) & \dots ② \end{cases}$$

→ 3 unknowns

→ 2 equations: Solutions: infinite ⟹ dimension of space of solution

none

some no.

focus of course.

It is a linear system:

Since x_1, x_2, x_3 have order of 1 (form: $c_1x_1 + c_2x_2 + c_3x_3 + \dots = c_4$)

Counterexamples:

$$\begin{cases} x_1^2 + e^{x_2} + 3x_3 = 1 \\ x_2x_3 = -7 \end{cases}$$

quadratic

to the power

non-linear

e.g. anything with unknowns $\{x_i\}$ multiplying or e^{x_i}, x_i^{-7}, $\cos(x_i)$ is never LINEAR

∵ There are non-linear operations,

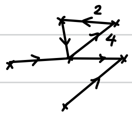
∴ System is non-linear

Applications: (Sample)

① Taylor series of a function starts with LINEAR APPROXIMATION

② Diagonalization: PageRank algorithm, powers of matrices

③ (MAT 145) graph theory



2. From equations to maps

① Why does this system:

$$\begin{cases} x_1 + 2x_2 = 4 \\ 3x_1 + 4x_3 = 7 \end{cases}$$ has a unique solution

BUT!

$$\begin{cases} x_1 + 2x_2 = 4 \\ 2x_1 + 4x_2 = 8 \end{cases}$$ has ∞ solutions

* You should understand the underlying structure of systems, instead of plugging & checking

Maps ⟹ well-define; Matrices ⟹ depend on POV (i.e. BASES)

∴ For every map, there are many matrices.

Strategy:

Linear system of eqn ⟷ Linear map $f: \mathbb{R} \to \mathbb{R}$ (# vars → # eqns) ⟷ matrix A_f of # eqn rows & # vars colums

CHOICE OF BASES

(reversible)

Def: A map $f: \overset{\text{\#vars}}{\mathbb{R}^n} \to \overset{\text{\#eqn}}{\mathbb{R}^m}$, $n,m \in \mathbb{N}$, is an assignment

$$\underbrace{(x_1, \dots, x_n)}_{\substack{\text{n-tuple of} \\ \mathbb{R}}} \longmapsto \underbrace{f(x_1, \dots, x_n)}_{\substack{\text{m-tuple of} \\ \mathbb{R}}}$$

INPUT OUTPUT

Examples: $n=1,\ m=1,\quad f: \mathbb{R} \to \mathbb{R},\ f(x_1) = (\cos(x_1)\cdot x_1^2)^{\frac{1}{2}} + e^{x_1}$

$n=2,\ m=1,\quad f: \mathbb{R}^2 \to \mathbb{R},\ f(x_1, x_2) = 3x_1 + 4x_2$

$n=2,\ m=2,\quad f: \mathbb{R}^2 \to \mathbb{R}^2,\ f(x_1, x_2) = (x_1 + 2x_2,\ 3x_1 + 4x_2)$

$\Rightarrow$ Solving (*) is asking about inputs of with output $(4,7)$

$$* \begin{cases} x_1 + 2x_2 = 4 \\ 3x_1 + 4x_3 = 7 \end{cases}$$