Math 16A (Fall 2007)
Kirkbride
Midterm 1

Please PRINT your name here :

Your Student ID Number:

1. PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO.
2. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO, IN ANY WAY, ASSIST ANOTHER PERSON IN THE COMPLETION OF THIS EXAM. PLEASE KEEP YOUR OWN WORK COVERED UP AS MUCH AS POSSIBLE DURING THE EXAM SO THAT OTHERS WILL NOT BE TEMPTED OR DISTRACTED. THANK YOU FOR YOUR COOPERATION.
3. No notes, books, or classmates may be used as resources for this exam.
4. Read directions to each problem carefully. Show all work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of your getting a good score on this exam. Neatness and organization are also important.
5. Make sure that you have 5 pages, including the cover page.
6. You may NOT use L'Hopital's Rule on this exam.
7. You may NOT use shortcuts for finding limits to infinity.
8. You will be graded for proper use of limit notation.
9. Put units on answers where units are appropriate.
10. You have until 3:00pm sharp to finish the exam.

1. Determine if the limit exists and evaluate (4pts each).

Plug in "$\frac{\infty}{\infty} + \frac{\infty}{\infty}$"

a) $\lim_{x\to\infty}\left(\frac{2x}{x^3-1}+\frac{x^2}{3x^2+1}\right) = \lim_{x\to\infty}\left(\frac{2x}{x^3-1}\right) + \lim_{x\to\infty}\left(\frac{x^2}{3x^2+1}\right) =$

$= \lim_{x\to\infty}\left(\frac{2x}{x^3-1}\right)\frac{1/x^3}{1/x^3} + \lim_{x\to\infty}\left(\frac{x^2}{3x^2+1}\right)\cdot\frac{1/x^2}{1/x^2} =$

$= \lim_{x\to\infty}\frac{\overbrace{2/x^3}^{\to 0}}{1-\underbrace{(1/x^3)}_{\to 0}} + \lim_{x\to\infty}\frac{1}{3+\underbrace{(1/x^2)}_{\to 0}} = \frac{0}{1} + \frac{1}{3} = 0 + \frac{1}{3} = \frac{1}{3}$

Plug in "$\frac{36}{0}$"

b) $\lim_{x\to 4^+}\frac{x^2+5x}{x^2-7x+12} = \lim_{x\to 4^+}\frac{x(x+5)}{(x-3)(x-4)}$

(Note: we know it has something to do with infinity)

Put in a number close to 4 from the right side, say 4.1

$\Rightarrow \lim_{x\to 4^+}\frac{\overset{+}{x}\overset{+}{(x+5)}}{\underset{+}{(x-3)}\underset{+}{(x-4)}} = +\infty$

c) $\lim_{x\to\frac{\pi}{2}}(\cos x - \sin x) = \cos\pi/2 - \sin\pi/2 = 0 - 1 = -1$

Plug in "$\frac{0}{0}$"

d) $\lim_{x\to -2}\frac{\frac{1}{2x}+\frac{1}{4}}{x+2} = \lim_{x\to -2}\frac{\frac{2}{4x}+\frac{x}{4x}}{x+2} = \lim_{x\to -2}\frac{(2+x)}{4x}\cdot\frac{1}{(x+2)}$

$= \lim_{x\to -2}\frac{1}{4x} = -\frac{1}{8}$

2. (4 pts) Find the points of intersection (algebraically) of the graphs of $y = -8x$ and $y = x^2$. If $y = -8x$ and $y = x^2$

$\Rightarrow \quad x^2 = -8x$

$x^2 + 8x = 0$

$x(x+8) = 0$

so either $x = 0$, $y = 0$ $\quad$ $x + 8 = 0$, $x = -8$, $y = 64$

3. (4 pts) Solve the trigonometric equation for θ for $0 \leq \theta < 2\pi$.

$$\cos^2\theta = \cos\theta$$

$\cos^2\theta - \cos\theta = 0$

$\cos\theta(\cos\theta - 1) = 0$

$\cos\theta = 0$ $\quad$ $\cos\theta - 1 = 0$

$\theta = \pi/2, 3\pi/2$ $\quad$ $\cos\theta = 1$

$\theta = 0$

4. (4 pts) The function $g(x) = \dfrac{3}{4 - 5x}$ is one-to-one. Find the inverse $g^{-1}(x)$.

$y = \dfrac{3}{4-5x}$

$x = \dfrac{3}{4-5y}$

$(4-5y)x = 3$

$4x - 5xy = 3$

$-5xy = 3 - 4x$

$y = -\dfrac{3-4x}{5x}$

5. (4 pts) Determine the value of the constant a so that the following is continuous for all values of x and give the resulting piecewise function $f(x)$.

$$f(x) = \begin{cases} ax^2 - 4 \text{ if } x < 2, \\ -x + 3a \text{ if } x \geq 2 \end{cases}$$

$\lim_{x \to 2^+} -x + 3a = -2 + 3a$

$\lim_{x \to 2^-} ax^2 - 4 = 4a - 4$

$\Rightarrow \quad -2 + 3a = 4a - 4$

$-3a \quad -3a$

$-2 = a - 4$

$2 = a$

6. Consider the functions $f(x) = x^2 - 2x + 3$ and $g(x) = \sin x - 5$.

a) (4 pts) Find and simplify the functional composition $f(g(x))$.

$$f(g(x)) = (\sin x - 5)^2 - 2(\sin x - 5) + 3$$
$$f(g(x)) = \sin^2 x - 10\sin x + 25 - 2\sin x + 10 + 3$$
$$f(g(x)) = \sin^2 x - 12\sin x + 38$$

b) (3 pts) Is $f(g(x))$ continuous on the entire real life? Why or why not?

The composition of continuous functions is continuous, so YES.

7. Consider the function $f(x) = x^2 + 3x$

a) (5 pts) Use the LIMIT DEFINITION of the derivative to find the derivative of this function.

$$f'(x) = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 + 3(x+\Delta x) - x^2 - 3x}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 + 3x + 3\Delta x - x^2 - 3x}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2 + 3\Delta x}{\Delta x} = \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(2x + \Delta x + 3)}{\cancel{\Delta x}}$$
$$= \lim_{\Delta x \to 0} 2x + \Delta x + 3 = 2x + 3$$

b) (3 pts) Find the derivative using another method and state whether your two answers are the same.

$$f(x) = x^2 + 3x$$
$$f'(x) = 2x + 3$$

Yes, they must be the same

8. (5 pts) Determine the x-value of the point at which the tangent line to $y = \sqrt{x} + 2x + 1$ is parallel to the line $y = 3x + 17$.

$y = x^{1/2} + 2x + 1$ $\qquad$ $y = 3x + 17$

slope $\quad y' = \frac{1}{2}x^{-1/2} + 2$ $\qquad$ $m = 3 \leftarrow$ slope

$\Rightarrow \quad \frac{1}{2\sqrt{x}} + 2 = 3$ $\qquad$ $\left(\frac{1}{2}\right)^2 = (\sqrt{x})^2$

$\frac{1}{2\sqrt{x}} = 1$ $\qquad$ $\frac{1}{4} = x$

$1 = 2\sqrt{x}$

9. Consider the function $f(x) = \dfrac{x-1}{2x^2 - 3x + 1}$

a) (4 pts) Use limits to find all vertical asymptotes

Plug in "$\frac{1}{0}$" $\quad \lim_{x \to 1/2^+} \frac{1}{2x-1} = +\infty$

Plug in "$\frac{1}{0}$" $\quad \lim_{x \to 1/2^-} \frac{1}{2x-1} = -\infty$

$x = 1/2$ is a vertical asymptote

b) (4 pts) Use limits to find all horizontal asymptotes

$$\lim_{x \to \pm\infty} \frac{x-1}{2x^2-3x+1} = \lim_{x \to \pm\infty} \frac{x-1}{2x^2-3x+1} \cdot \frac{1/x^2}{1/x^2}$$

$$= \lim_{x \to \pm\infty} \frac{\overset{\to 0}{1/x} - \overset{\to 0}{1/x^2}}{2 - \underset{\to 0}{3/x} + \underset{\to 0}{1/x^2}} = \frac{0}{2} = 0$$

$y = 0$ is a horizontal asymptote