

Math 16A Short Calculus, section 3
Practice Midterm, Part 1

1. **True/False**

Mark each question as (**T**)rue or (**F**)alse.

- a. ___ The line $(y - 3) = 2(x + 4)$ is in point-slope form.
- b. ___ If $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x^2}$, then $f \circ g(x) = \frac{1}{x^3}$.
- c. ___ If $p(x)$ is a polynomial function and c is any real number, then $\lim_{x \rightarrow c} p(x) = p(c)$.
- d. ___ Two lines are parallel if and only if their slopes are negative reciprocals of each other.
- e. ___ All continuous functions are differentiable.
- f. ___ The graph of $y = \frac{1}{x^2+5}$ has a vertical asymptote.
- g. ___ Infinity is a real number.
- h. ___ The equation $x^2 + 2y^2 = 48$ represents a circle.
- i. ___ The graph of $y = 2x^2 + 2$ is symmetrical in the y -axis.
- j. ___ If an equation passes the horizontal line test, then the equation defines a function.

2. Lines and Circles

(a) Consider the points $(2, -3)$ and $(-2, 3)$.

i. What is the distance between the points?

ii. What is their midpoint?

iii. Write the line containing them in slope-intercept form.

(b) Consider the circle whose general form is

$$3x^2 + 3y^2 - 6x + 18y - 18 = 0.$$

i. Write the equation in standard form.

ii. State the center, radius, and x -intercepts.

3. Functions

Let $f(x) = x^2 - 2$ and $g(x) = \sqrt{x - 1}$.

(a) Sketch the graphs of each function, labelling intercepts.

(b) State an appropriate domain for $g(x)$. What is the range of $g(x)$?

(c) Compute the following:

i. $f(x) - g(x)$

ii. $f(x)/g(x)$

iii. $g \circ f(x)$

(d) Find $g^{-1}(x)$. What is its domain?

4. Limits

Evaluate the following limits.

(a) $\lim_{x \rightarrow 3} 3x + 3$

(b) $\lim_{x \rightarrow -1} \frac{x^2 - 1}{x + 1}$ [HINT: Factorize the numerator.]

(c) $\lim_{x \rightarrow 3^+} \frac{|x - 3|}{x - 3}$

5. Continuity

(a) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function and let a be a real number. What does it mean to say that f is continuous at a ?

(b) Give an example of a function for which $\lim_{x \rightarrow 3} f(x)$ exists but the function is not continuous at 3. Carefully sketch the graph of your function.

6. Differentiation

(a) Find the equation of the tangent to $y = x^3$ at $(3, 9)$.