Math 21C

Chapter 14 Problems

1. Describe and sketch the domain of each of the following functions.

(a) $f(x, y) = \sqrt{xy}$ (b) $f(x, y) = \sqrt{(1 - x^2)(y^2 - 4)}$

2. Find the limit or show that it does not exist.

(a) $\lim_{(x,y)\to(0,0)} \dfrac{x - y}{x + y}$ (b) $\lim_{(x,y)\to(0,0)} \dfrac{(x + 1)^2 - (y + 1)^2}{(x - 1)^2 - (y - 1)^2}$

3. Find the parametric equations of the tangent line to the curve

$$x = t^2 - 1, y = t^3, z = t^4$$

at the point $(0, -1, 1)$.

4. Find partial derivatives $\dfrac{\partial z}{\partial u}, \dfrac{\partial z}{\partial v}$ for given functions $z = z(x, y)$ using the given expressions of x and y as functions of u and v. Express the derivatives as functions of u and v.

(a) $z = \sin(xy),\ x = ue^v, y = ue^{-v}$ (b) $z = \ln\left|\dfrac{x - y}{x + y}\right|,\ x = u^2 + v^2, y = 2uv$

5. Find the direction of the fastest growth of the function

(a) $x^2 + 2y^2 + 3z^2$ at the point $(6, 3, 1)$ (b) $e^{x+y}\cos(x - y)$ at the point $(0, 0)$

and find the directional derivative of the given function at the given point in the direction of the fastest growth.

6. Find the equation of the tangent plane to the surface $z = x^2 - y^2$ at the point $(3, 2, 5)$.

7. Find all local maxima, local minima and saddle points for the function

$$f(x, y) = x^2 + xy + y^2 + \frac{1}{x} + \frac{1}{y},\ x > 0, y > 0.$$

8. Find all local maxima, local minima and saddle points for the function

$$f(x, y) = x^4 + y^4 - (x + y)^2.$$

9. Find all local maxima, local minima and saddle points for the function

$$f(x, y) = x^2 + axy + y^2,\ a \neq \pm 2$$

(dependingly on a).

10. Find the maximum and minimum values of the function $f(x, y) = x^2 + xy + y^2$ under the constraint $x^2 - xy + y^2 = 1$.

11. Find the maximum and minimum values of the function $f(x, y, z) = xyz$ on the sphere $x^2 + y^2 + z^2 = 3$.