11.6, Problem # 50. Since

$$\frac{1}{8!} \approx 0.0000248 = 2.48 \cdot 10^{-5} > 5 \cdot 10^{-6},$$
$$\frac{1}{9!} \approx 0.00000275 = 2.75 \cdot 10^{-6} < 5 \cdot 10^{-6},$$

the required precision will be provided by the partial sum$\displaystyle\sum_{n=0}^{8}(-1)^n \frac{1}{n!}$.

11.7, Problem # 28. Since

$$\lim_{n\to\infty}\left[\frac{1}{(n+1)\ln(n+1)} \div \frac{1}{n\ln n}\right] = \lim_{n\to\infty}\frac{n\ln n}{(n+1)\ln(n+1)} = 1,$$

the radius of convergence is 1. For $x = 1$, the series is $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n\ln n}$; it diverges by the integral test. For $x = -1$, the series is $\displaystyle\sum_{n=2}^{\infty}(-1)^n\frac{1}{n\ln n}$; it converges by the alternating series test. Thus, the given series converges absolutely for $-1 < x < 1$, converges conditionally for $x = -1$, and diverges for all other values of x.

11.7, Problem # 34. This is a geometric series, it converges, if and only if $\left|\frac{(x+1)^2}{9}\right| < 1$, that is, $-3 < x+1 < 3$, that is $-4 < x < 2$, and for these values of x the convergence is absolute. The sum is

$$\frac{1}{1-\frac{(x+1)^2}{9}} = \frac{9}{9-(x+1)^2} = \frac{9}{(2-x)(4+x)}$$

11.7, Problem # 36. This is a geometric series, it converges, if and only if $|\ln x| < 1$, that is, $\frac{1}{e} < x < e$. The sum is $\frac{1}{1-\ln x}$.

11.8, Problem # 6. Since $f'(x) = -\sin x,\ f''(x) = -\cos x, f'''(x) = \sin x,$

$$f\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2},\ f'\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2},\ f''\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2},\ f'''\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2},$$

and the Taylor polynomial of order 3 is

$$\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right) - \frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)^2 + \frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)^3;$$

the Taylors polynomials of orders 0,1, and 2 are obtained from this by truncating at the first, second and third terms.

11.8, Problem # 10. Put $y = \frac{x}{2}$. Then

$$e^y = \sum_{n=0}^{\infty} \frac{y^n}{n!}, \quad e^{\frac{x}{2}} = \sum_{n=0}^{\infty} \frac{\left(\frac{x}{2}\right)^n}{n!} = \sum_{n=0}^{\infty} \frac{x^n}{2^n n!};$$

this is the Maclaurin series for $e^{\frac{x}{2}}$.

11.8, Problem # 18. Since $f'(x) = 4x^3 - 6x^2 - 5,\ f''(x) = 12x^2 - 12x,\ f'''(x) = 24x - 12,\ f^{(4)}(x) = 24$, and the further derivatives of f are zeroes,

$$f(0) = 4,\ f'(0) = -5,\ f''(0) = 0,\ f'''(0) = -12,\ f^{(4)}(0) = 24,$$

and the Maclaurin series is

$$4 - 5x + \frac{0}{2!}x^2 - \frac{12}{3!}x^3 + \frac{24}{4!}x^4 = 4 - 5x - 2x^3 + x^4$$

(a polynomial is its own Maclaurin series).

11.8, Problem # 20. Since $f'(x) = 3x^2 - 2,\ f''(x) = 6x,\ f'''(x) = 6$, and the further derivatives of f are zeroes,

$$f(2) = 8,\ f'(2) = 10,\ f''(2) = 12,\ f'''(2) = 6,$$

and the Taylor series is

$$8 + 10(x-2) + 6(x-2)^2 + (x-2)^3.$$

11.8, Problem # 22. Since $f'(x) = 6x^2 + 2x + 3,\ f''(x) = 12x + 2,\ f'''(x) = 12$, and the further derivatives of f are zeroes,

$$f(1) = -2,\ f'(1) = 11,\ f''(1) = 14,\ f'''(1) = 12,$$

and the Taylor series is

$$-2 + 11(x-1) + 7(x-1)^2 + 2(x-1)^3.$$