

**12.1, Problem # 10.**  $x^2 + y^2 + z^2 = 25$  is the sphere centered at  $(0, 0, 0)$  of radius 5. The plane  $y = -4$  (parallel to the  $xz$ -plane) intersects this sphere in a circle. The center of this circle is  $(0, -4, 0)$ , the radius of the circle is  $\sqrt{5^2 - 4^2} = 3$ .

**12.1, Problem # 16.** Begin with (c).

(c)  $x^2 + y^2 \leq 1$  is a solid cylinder whose axis is the  $z$ -axis and whose radius is 1.

(a)  $x^2 + y^2 \leq 1$ ,  $z = 0$  is the intersection of the cylinder in (c) with the plane  $z = 0$  which is the  $xy$ -plane. This intersection is the disc in the  $xy$ -plane centered at  $(0, 0)$  of radius 1.

(b) Same, but in the plane  $z = 3$  which is a horizontal plane 3 units above the  $xy$ -plane.

**12.1, Problem # 20.** (a)  $x = 3$ ; (b)  $y = -1$ ; (c)  $z = 2$ .

**12.1, Problem # 22.** (a)  $x^2 + y^2 = 4$ ,  $z = 0$ ; (b)  $y^2 + z^2 = 4$ ,  $x = 0$ ; (c)  $x^2 + z^2 = 4$ ,  $y = 0$ .

**12.1, Problem # 44.** Center  $\left(0, -\frac{1}{3}, \frac{1}{3}\right)$ , radius  $\sqrt{\frac{29}{9}}$ .

**12.1, Problem # 50.** The equation

$$x^2 + y^2 + z^2 - 6y + 8z = 0$$

can be transformed to

$$x^2 + (y - 3)^2 + (z + 4)^2 = 25;$$

this is a sphere centered at  $(0, 3, -4)$  of radius 5.

**12.2, Problem # 20.**  $\overrightarrow{AB} = \langle -1 - 1, 4 - 0, 5 - 3 \rangle = \langle -2, 4, 2 \rangle = -2\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$ .

**12.2, Problem # 22.**  $-2\mathbf{u} + 3\mathbf{v} = -2\langle -1, 0, 2 \rangle + 3\langle 1, 1, 1 \rangle = \langle 5, 3, -1 \rangle = 5\mathbf{i} + 3\mathbf{j} - \mathbf{k}$

**12.2, Problem # 34.** Any vector whose direction is opposite to that of  $\frac{1}{2}\mathbf{i} - \frac{1}{2}\mathbf{j} - \frac{1}{2}\mathbf{k}$  has the form  $-a\mathbf{i} + a\mathbf{j} + a\mathbf{k}$  with  $a > 0$ . The length (magnitude)  $\sqrt{a^2 + a^2 + a^2} = a\sqrt{3}$  should be 3; hence  $a = \sqrt{3}$ , and our vector is  $-\sqrt{3}\mathbf{i} + \sqrt{3}\mathbf{j} + \sqrt{3}\mathbf{k}$ .

**12.2, Problem # 40.** If  $\overrightarrow{AB} = -7\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}$ ,  $A = (-2, 3, 6)$ , and  $B = (a, b, c)$ , then  $-7 = a + 2$ ,  $3 = b - 3$ ,  $8 = c - 6$ , hence  $a = -9$ ,  $b = 6$ ,  $c = 14$ , and  $B = (-9, 6, 14)$ .

**12.3, Problem # 4.** If  $\mathbf{v} = 2\mathbf{i} + 10\mathbf{j} - 11\mathbf{k}$ ,  $\mathbf{u} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ , then

(a)  $\mathbf{v} \cdot \mathbf{u} = 2 \cdot 2 + 10 \cdot 2 - 11 \cdot 1 = 13$ ,  $|\mathbf{v}| = \sqrt{2^2 + 10^2 + 11^2} = 15$ ,  $|\mathbf{u}| = \sqrt{2^2 + 2^2 + 1^2} = 3$ , (b)  $\cos \theta = \frac{13}{15 \cdot 3} = \frac{13}{45}$ , (c) the scalar component of  $\mathbf{u}$  in the direction of  $\mathbf{v}$  is  $|\mathbf{u}| \cdot \cos \theta = 3 \cdot \frac{13}{45} = \frac{13}{15}$ , (d)  $\text{proj}_{\mathbf{v}} \mathbf{u} = \frac{\mathbf{v} \cdot \mathbf{u}}{|\mathbf{v}|^2} \mathbf{v} = \frac{13}{225} \mathbf{v} = \frac{26}{225} \mathbf{i} + \frac{130}{225} \mathbf{j} - \frac{143}{225} \mathbf{k}$

**12.3, Problem # 10.**

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}| \cdot |\mathbf{v}|} = \frac{2 \cdot 3 - 2 \cdot 0 + 1 \cdot 4}{3 \cdot 5} = \frac{10}{15} = \frac{2}{3};$$

$$\theta \approx 0.84 \text{ rad.}$$