Practice Midterm # 2

These problems are given in addition to the collection of problems entitled "Planes and lines in space".

1. What do the following equation and inequalities describe in space?

(a) $x^2 + y^2 + z^2 - 8x + 6z = 0$

(b) $x^2 + y^2 + z^2 - 8x + 6z \leq 0$

(c) $x^2 + y^2 + z^2 - 8x + 6z \geq 0$

2. Let $\mathbf{u}, \mathbf{v}$ be two vectors, and let $\mathbf{u} - 2\mathbf{v} = 2\mathbf{v} - 3\mathbf{u}$. Prove that $\mathbf{u} = \mathbf{v}$.

3. Let $\mathbf{u} = \mathbf{i} - 2\mathbf{k}$, $\mathbf{v} = \mathbf{j} - 2\mathbf{i}$. Find the projections $\text{proj}_{\mathbf{u}}\mathbf{v}$, $\text{proj}_{\mathbf{v}}\mathbf{u}$.

4. Let $\mathbf{u} = \mathbf{i} - \mathbf{k}$, $\mathbf{v} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{w} = \mathbf{j} - \mathbf{i}$. Find all pairs of perpendicular vectors among $\mathbf{u}, \mathbf{v}, \mathbf{w}$.

5. Find the angle between the vectors $\mathbf{i} + \mathbf{k}$ and $\mathbf{j} - \mathbf{k}$.

6. Find a vector perpendicular to the vectors $\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ and $\mathbf{j} - \mathbf{k}$.

7. Given $\mathbf{u} \times \mathbf{v} = \mathbf{w}$. Find $(2\mathbf{u} + 3\mathbf{v}) \times (\mathbf{v} - \mathbf{u})$.

8. A particle moves in space, and at every moment t it has coordinates $(t, t^2 - t, t^3 - t^2)$. Find the velocity vector and the acceleration vector at the moment $t = 0$.

9. A particle moves in space, and at every moment t its acceleration vector is $\langle 1, t - 1, t^2 \rangle$. At the moment $t = 0$, the particle has coordinates $(0, 1, 2)$, and its velocity vector is $\langle 2, 1, 0 \rangle$. Find the coordinates of the particle at the moment $t = 1$.

10. A curve has parametric equations $x(t) = 1 + t$, $y(t) = t^2$, $z(t) = \frac{2}{3}t^3$. Find the length of the segment of the curve between the points $(1, 0, 0)$ and (4,9,18).