

Practice Midterm # 1

1. Let S be the series $\sum_{n=0}^{\infty} \left(\frac{x}{1+x}\right)^n$ where $x \neq -1$.

- Determine the values of x for which S converges. Justify your answer.
- Find all values of x that make the sum of the series S greater than 10.

Determine whether the following series converge or diverge. Justify your answer.

2. $\sum_{n=1}^{\infty} \frac{5 \cdot 7^n + 7 \cdot 5^n}{12^n}$

3. $\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^n$

4. $\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n^2}$

5. $\sum_{n=0}^{\infty} \frac{(2n+3)^2}{(n+1)^3}$

6. $\sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{1}{n}$

7. $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$

8. How many terms of the series do we need to add in order to find the sum $\sum_{n=1}^{\infty} \frac{(-1)^n n}{4^n}$ with an error less than 0.002?

Determine whether the following series absolutely converge, conditionally converge or diverge. Justify your answer.

9. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \ln n}{n}$

10. $\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^n}$

11. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{n^2 + 1}$

12. $\sum_{n=0}^{\infty} \frac{(-5)^n}{2^{2n+1}}$

For which values of x do the following series absolutely converge, conditionally converge or diverge. Justify your answer.

13. $\sum_{n=0}^{\infty} \left(\frac{e^n}{n+1}\right) x^n$

14. $\sum_{n=0}^{\infty} (-1)^n \frac{4^{2n}}{\sqrt{n+1}} x^n$

15. $\sum_{n=1}^{\infty} \frac{(x+2)^n}{\sqrt[4]{n}}$

16. $\sum_{n=1}^{\infty} \frac{n}{1+n^2} (x-2)^n$

17. Find the sum of the convergent series

$$\ln 3 - \frac{(\ln 3)^2}{2!} + \frac{(\ln 3)^3}{3!} - \frac{(\ln 3)^4}{4!} + \dots$$