Math 21C

Quiz #3: solutions

1. Does the series

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^2+1}{n^3+2n+1}$$

diverge, conditionally converge, or absolutely converge? Justify your answer.

Solution. This is an alternating series, it converges by the alternating series test: the sequence $\left\{\frac{n^2+1}{n^3+2n+1}\right\}$ decreases and has zero limit. The series of absolute values, $\sum_{n=1}^{\infty}\frac{n^2+1}{n^3+2n+1}$ diverges by limit comparison test:

$$\lim_{n\to\infty}\left[\frac{n^2+1}{n^3+2n+1}\div\frac{1}{n}\right]=\lim_{n\to\infty}\frac{n^3+1}{n^3+2n+1}=1.$$

Thus, our series converges conditionally.

2. Prove that the series

$$\sum_{n=1}^{\infty}\left(-\frac{1}{n}\right)^n$$

converges. What is the minimal number of terms one need to take to approximate the sum with the error less than $\frac{1}{100}$? Justify your answer.

Solution. This is an alternating series, it converges by the alternating series test. The first term less than $\frac{1}{100}$ is $\left(\frac{1}{4}\right)^4=\frac{1}{256}$. Thus we need to take 3 terms: $1-\frac{1}{4}+\frac{1}{27}$.