

Let

$$\mathbf{u} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$$

$$\mathbf{v} = \mathbf{i} + 3\mathbf{j} - \mathbf{k}$$

1. Find the length of the vector $2\mathbf{u} + 3\mathbf{v}$.

Solution.

$$2\mathbf{u} + 3\mathbf{v} = (2 \cdot 2 + 3 \cdot 1)\mathbf{i} + (2 \cdot (-1) + 3 \cdot 3)\mathbf{j} + (2 \cdot 3 + 3 \cdot (-1))\mathbf{k} = 7\mathbf{i} + 7\mathbf{j} + 3\mathbf{k};$$

$$|2\mathbf{u} + 3\mathbf{v}| = \sqrt{7^2 + 7^2 + 3^2} = \sqrt{107}.$$

2. Find the cosine of the angle between the vectors \mathbf{u} and \mathbf{v} .

Solution.

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}| \cdot |\mathbf{v}|} = \frac{2 \cdot 1 + (-1) \cdot 3 + 3 \cdot (-1)}{\sqrt{2^2 + 1^2 + 3^2} \cdot \sqrt{1^2 + 3^2 + 1^2}} = -\frac{4}{\sqrt{154}}.$$