

1. Let $\mathbf{u} = \mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{v} = \mathbf{i} - \mathbf{j} + \mathbf{k}$. Find $(\mathbf{u} \cdot \mathbf{v})(\mathbf{u} \times \mathbf{v})$.

Solution.

$$\mathbf{u} \cdot \mathbf{v} = 1 \cdot 1 + 1 \cdot (-1) + (-1) \cdot 1 = 1 - 1 - 1 = -1.$$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{vmatrix} = -2\mathbf{j} - 2\mathbf{k}.$$

$$(\mathbf{u} \cdot \mathbf{v})(\mathbf{u} \times \mathbf{v}) = (-1)(-2\mathbf{j} - 2\mathbf{k}) = 2\mathbf{j} + 2\mathbf{k}.$$

2. Let $P = (1, 0, 0)$, $Q = (0, 1, 0)$, $R = (0, 0, 1)$. Find an equation of the plane passing through the point P and perpendicular to the line QR .

Solution. $\overrightarrow{QR} = (0, -1, 1)$. Thus, the plane has an equation $0 \cdot x + (-1) \cdot y + 1 \cdot z + D = z - y + D = 0$. Plug P to find D : $0 - 0 + D = 0$, $D = 0$. Answer: $z - y = 0$.