Math 21C **Quiz #7, solutions**

1. For the function $f(x,y,z) = x^2 + yz$, find the direction of the fastest growth at the point $(1,2,1)$. What is the derivative of the given function at the given point in the direction of the fastest growth?

Solution. The partial derivatives of the given function are, correspondingly, $2x$, z and y. The values of these derivatives at the given point are the components of the gradient. Hence, $\nabla_{(1,2,1)} f = \langle 2,1,2 \rangle$. The direction of the fastest growth is the direction of the gradient,

$$\frac{\nabla_{(1,2,1)} f}{\left|\nabla_{(1,2,1)} f\right|} = \frac{\langle 2,1,2 \rangle}{\sqrt{2^2+1^2+2^2}} = \left\langle \frac{2}{3}, \frac{1}{3}, \frac{2}{3} \right\rangle.$$

The derivative in the direction of the fastest growth is the length of the gradient, that is, 3. (It also can be found directly from the definition of the directional derivative: $\frac{2}{3} \cdot 2 + \frac{1}{3} \cdot 1 + \frac{2}{3} \cdot 2 = 3$.)

2. Write the equation of the tangent plane to the surface $z - x^2 - 2y^2 = 0$ at the point $(1,1,3)$.

Solution. The gradient of the function $z - x^2 - 2y^2$ at the point $(1,1,3)$ is $\langle -2 \cdot 1, -4 \cdot 1, 1 \rangle = \langle -2,-4,1 \rangle$. Thus, the equation of the tangent plane is $-2x - 4y + z + D = 0$ where D can be found from the condition that the point $(1,1,3)$ lies on the plane; thus, $D = 3$, and the equation is $-2x - 4y + z + 3 = 0$.