

**Math and Computers, Math 165**  
**Final review**

1. Exam will cover, from the textbook, Entire chapter 1, Chapter 2 all except Section 2.9. Chapter 6 section 4. The material on univariate polynomials we saw about Descartes rule of signs, Sturm sequences, etc is also included. Make sure to review all key theorems and know when they can be applied.
2. Find the gcd of  $x^2 + 3x - 4$ ,  $x^3 + 2x^2 - 7x - 14$ , and  $x^3 - 6x^2 + x + 4$ .
3. Using what you have learned find as much information as you can about the possible number of roots (counting multiplicities) of the following polynomial:  $x^9 - x^5 + 2x^3 - x^2 + x - 1$
4. Using 3 different monomial orders write the monomials of the following polynomial  $x^4y^5z + 2x^3y^2z - 4xyz^4 + x^2z^3 - x^2y^2$ .
5. State's Dickson's lemma. Use it to prove Hilbert's basis theorem.
6. Is the variety  $V(\{x^4y^2 - z^5, x^3y^3 - 1, x^2y^4 - 2z\})$  finite? what is its dimension?
7. Solve the system of equations  $x^2 + y^2 + z^2 = 4, x^2 + 2y^2 = 5, xz = 1$ .