Math and Computers, Math 165
Homework four, Due November 13

1. Write your own MAPLE implementation of the multivariate division algorithm using lexicographic and graded lexicographic orders. Check your answer computing the division of $x^7y^2 + x^3y^2 - y + 1$ by $[xy^2 - x, x - y^3]$. HINT: use MAPLE's commands to handle monomial orders.

2. Using graded lex order, find an element of the ideal $< f_1, f_2 >=< 2xy^2 - x, 3x^2y - y - 1 >$ whose remainder on division by $[f_1, f_2]$ is non-zero.

3. How many monomials in variables $x_1, x_2, \ldots, x_n$ have total degree m.

4. Problem Chapter 2.4 number 8.

5. Show that for each n there exist a monomial ideal $I \subset K[x, y]$ such that every basis of I has at least n elements.

6. Compute the S-pairs of the following polynomials using the lexicographic monomial order.

   - $f = 4x^2z - 7y^2$, $g = xyz^2 + 3xz^4$.
   - $f = x^4y - z^2$, $g = 3xz^2 - y$
   - $f = xy + z^3$, $g = z^2 - 3z$.