

**Math and Computers, Math 165**  
**review problems**

1. Using Lagrange multipliers find the point(s) on the surface  $x^4 + y^2 + z^2 - 1 = 0$  closest to the point  $(1, 1, 1)$ .
2. Using Gröbner bases find all critical points of the polynomial function (check your calculus book for the definition).

$$f(x, y) = (x^2 + y^2 - 4)(x^2 + y^2 - 1) + \left(x - \frac{3}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2$$

3. What is the Buchberger criterion, show an example.
4. Find all solutions of the system of equations  $x^2 + y^2 + z^2 - 1 = 0$ ,  $x^2 + y^2 + z^2 - 2x = 0$ ,  $2x - 3y - z = 0$ .
5. State the Hilbert Nullstellensatz and show why it is important for Automatic theorem proving.