Enumerative Combinatorics, Math 245
Homework two

1. Problems from Stanley's book Chapter 1: 23 (a,c), 30(a,b), 34, 43, 44
2. If a permutation $\pi = a_1a_2...a_n$ has an inversion vector $(b_1, b_2, ..., b_n)$ what is the permutation that corresponds to the inversion vector $(n-1-b_1, n-2-b_2, ..., 0-b_n)$?
3. Find an explicit formula for a_n if $a_0 = 0$ and $a_1 = 1$ and if for all integers $n \geq 2$ we have $a_n = 4a_{n-1} - 5a_{n-2}$.
4. Using generating functions find an explicit formula for a_n if it satisfies the recurrence $a_n = na_{n-1} + (-1)^n$ and $a_0 = 1$.
5. What is the number of permutations in S_n so that there is no triple $i < j < k$ with $\pi(j) < \pi(i) < \pi(k)$?
6. Verify that the ring of formal power series is an integral domain. What is its quotient field?
7. Prove that the number of labeled spanning trees on the complete graph on n nodes is n^{n-2}.
8. The Bell number B_n is the number of all partitions of the set $[n]$. Find a recurrence relation for B_n then prove that the exponential generating function of B_n is

$$p(x) = \sum_{n=0}^{\infty} \frac{B_n}{n!} x^n = e^{e^x - 1}$$