MAT22B — HW #7

Due: Friday, 22 May.

In problems 1–7, solve the initial value problem.

(1) $\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix} \mathbf{x}\,, \qquad \mathbf{x}(0) = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

(2) $\mathbf{x}' = \begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ 3 & 3 & -1 \end{pmatrix} \mathbf{x}\,, \qquad \mathbf{x}(0) = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$

(3) $\mathbf{x}' = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & 1 \\ 1 & 10 & 2 \end{pmatrix} \mathbf{x}\,, \qquad \mathbf{x}(0) = \begin{pmatrix} 1 \\ -4 \\ 13 \end{pmatrix}$

(4) $\mathbf{x}' = \begin{pmatrix} 3 & 1 & -2 \\ -1 & 2 & 1 \\ 4 & 1 & -3 \end{pmatrix} \mathbf{x}\,, \qquad \mathbf{x}(0) = \begin{pmatrix} 1 \\ 4 \\ -7 \end{pmatrix}$

(5) $\mathbf{x}' = \begin{pmatrix} 1 & -1 \\ 5 & -3 \end{pmatrix} \mathbf{x}\,, \qquad \mathbf{x}(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

(6) $\mathbf{x}' = \begin{pmatrix} -3 & 0 & 2 \\ 1 & -1 & 0 \\ -2 & -1 & 0 \end{pmatrix} \mathbf{x}\,, \qquad \mathbf{x}(0) = \begin{pmatrix} 0 \\ -1 \\ -2 \end{pmatrix}$

(7) $\mathbf{x}' = \begin{pmatrix} 0 & 2 & 0 & 0 \\ -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 3 & 0 \end{pmatrix} \mathbf{x}\,, \qquad \mathbf{x}(0) = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$

(8) Consider the following 2nd order homogeneous linear differential equation:

$$y'' - 4y' + 3y = 0$$

The goal of this problem is understand how our old way of solving such an equation is related to the matrix methods we're studying now. Find the general solution in two ways:

(a) Solve it using the methods we studied in Chapter 3.

(b) By defining $u = y'$, show that the 2nd order linear equation can be written as 1st order linear equation:

$$\mathbf{x}' = \begin{pmatrix} 0 & 1 \\ -3 & 4 \end{pmatrix} \mathbf{x} \quad \text{where we define } \mathbf{x} = \begin{pmatrix} y \\ u \end{pmatrix}.$$

Solve this equation and compare your result to the answer from part (a).