# MAT22B — HW #8

**Due: Wednesday, 3 June.**

(1) Find 3 linearly independent solutions of $\mathbf{x}' = A\mathbf{x}$, where $A = \begin{pmatrix} 1 & 2 & -3 \\ 1 & 1 & 2 \\ 1 & -1 & 4 \end{pmatrix}$

(2) Solve the initial value problem:
$$\mathbf{x}' = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \mathbf{x}\,, \qquad \mathbf{x}(0) = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

(3) Given $A = \begin{pmatrix} -3 & 2 \\ -1 & -1 \end{pmatrix}$, find the matrix $e^{At}$.

(4) Recall that the differential equation $y' + by = g(t)$ can be solved using an integrating factor $\mu(t) = e^{bt}$. Now suppose we generalize this to
$$\mathbf{x}' + B\mathbf{x} = \mathbf{g}(t)$$
where $B$ is an $n \times n$ matrix and $\mathbf{g}(t)$ is a vector-valued function. Show that this equation can be solved using an integrating factor $e^{Bt}$. In particular, show that the solution is:
$$\mathbf{x}(t) = e^{-Bt} \int e^{Bt}\mathbf{g}(t)\,dt$$

(5) Use the result of the previous problem to solve the nonhomogeneous differential equation
$$\mathbf{x}' + \begin{pmatrix} -3 & 2 \\ -1 & -1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} e^{2t} \\ 3e^{2t} \end{pmatrix}$$