Math 21C **Homework #0** **due 6/23/08**

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1. Prove part 2 of theorem 1 on page 735 of the text. You may use other parts of the same theorem.

2. Provide an example of two divergent series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ such that the series $\sum_{n=1}^{\infty}(a_n + b_n)$ converges.

3. $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges, but $\sum_{n=1}^{\infty} \frac{-1}{n}$ diverges.
Explain why this does not contradict the comparison test.

1. Part 2 of theorem 1 states that if
$\{a_n\}$ and $\{b_n\}$ are sequences of real numbers, A and B are real numbers, and both $a_n \to A$ and $b_n \to B$, then $(a_n - b_n) \to (A - B)$.

We will use parts 1 and 4 of the same theorem to prove this. Note,

$a_n - b_n = a_n + (-1)b_n$. Then by part 4 of theorem 1,

$(-1)b_n \to (-1)B$. Then by part 1 of theorem 1,

$(a_n + (-1)b_n) \to (A + (-1)B)$. Hence,

$(a_n - b_n) = (a_n + (-1)b_n) \to (A + (-1)B) = A - B$.

2. Let $a_n = n$ and $b_n = -n$. Note,

$a_n \to \infty$ and $b_n \to -\infty$. Then by the *nth term test*, both

$\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ diverge. Note, however, that

$a_n + b_n = n - n = 0$. Then $\sum_{n=1}^{\infty} a_n + b_n = \sum_{n=1}^{\infty} 0 = 0$ clearly converges.

3. The *comparison test* states that if $\sum_{n=1}^{\infty} a_n$ is a series with no negative terms, then the following statement is true:

If there is some N such that for all $n > N$, $a_n \leq c_n$ and $\sum_{n=1}^{\infty} c_n$ converges, then $\sum_{n=1}^{\infty} a_n$ also converges.

It is certainly true that for all $n > 0$,

$\frac{-1}{n} < \frac{1}{n^2}$. It would then seem that since $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges,

$\sum_{n=1}^{\infty} \frac{-1}{n}$ should also converge. But the sequence $\frac{-1}{n}$ has negative terms, so the theorem does not even apply. Thus, this does not contradict the comparison test.