Math 21C **Homework #10** **due 7/28/08**

1. Let $f(x, y)$ be a smooth function purely of $x^2 + y^2$, i.e.
$f(x, y) = g(x^2 + y^2)$.

Prove that $y\frac{\partial f}{\partial x} - x\frac{\partial f}{\partial y} = 0$.

2. A California gold miner's profit, p, is determines by three variables: the number of hours mining per day, h, the number of ore veins at the day's site, v, and the quality of the ore found, q. The profit is given by the equation
$p(h, v, q) = 2hv + 3hq + 1vq + hvq$.
The miner is currently working 2 hours per day at a site with 6 veins, at quality $\frac{1}{2}$. In what direction in $hvq-$space should the miner move to increase his profits the fastest?

3. Let $f(x, y, z) = c$ define a smooth surface in three-dimensional space. Prove that the normal vector field to the surface is given by ∇f.

4. As a special case of the above, show that the normal vector to the plane $ax + by + cz = d$ is given by $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$.

5. Generalize your proof in #3 to an arbitrary number of finite dimensions.

6. Could you extend this to an infinite-dimensional case? Why or why not?

7. Recall, the *linearization* of a function (possibly of more than one variable) is the first-order Taylor approximation for the function. Write a formula analagous to that on page 1003 of your text for the *quadratization*, i.e. the second-order Taylor approximation. Can you continue to third- or higher-order terms? (Hint: the answer to the final question is "yes, but it's hard")

8. Explain in words why studying quadric surfaces is sufficiently general to learn most of the important details in multivariate calculus. (Hint: your quadrization or higher-order expansions may help)