# Math 21C
# Midterm 1
7/3/08

**Name**:

**Signature**:

**Student ID**:

- There are **ten** (plus cover and bonus) pages to the exam.
- The exam totals 100 points, plus 10 bonus points.
- You will have 50 minutes to complete the exam.
- No calculators, notes, or books allowed.
- Good luck!

| Problem | Points | Your Score |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 10 | |
| 6 | 10 | |
| 7 | 10 | |
| 8 | 10 | |
| 9 | 10 | |
| 10 | 10 | |
| Bonus | 10 | |
| Total | 100 | |

**1**. Definitions and Examples:

a. (2 points) Write the definition of a *Geometric Series*. (an example is **not** sufficient for full credit)

b. (2 points) Write the definition of an *Alternating Series*. (an example is **not** sufficient for full credit)

c. (2 points) Write the definition of a *p-series*. (an example is **not** sufficient for full credit)

d. (2 points) Write the definition of a *power series centered at* $c$. (an example is **not** sufficient for full credit)

e. (2 points) Let $f = f(x)$. Write the definition of the *Taylor series* for $f$ centered at $c$. (an example is **not** sufficient for full credit)

**2**. Short Answers

a. (5 points) State the *ratio test* for infinite series.

b. (5 points) State the *nth-term test* for infinite series.

**3**. Determine whether the following sequences converge or diverge. If a sequence converges, find its limit.

a. (3 points) $a_n = \frac{n+1}{n-1}$.

b. (3 points) $b_n = 1 + e^{\frac{\sin n}{n}}$.

c. (4 points) $a_n = \frac{(2n-1)!}{2(n-1)!}$.

**4**. Determine whether the following series converge or diverge. If a geometric series converges, find its limit. If convergence depends on a constant input, provide conditions on this constant.

a. (5 points) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{p^n}.$

b. (5 points) $\displaystyle\sum_{n=1}^{\infty} c - \frac{n}{n+1}.$

**5**. (10 points) Determine the values of $x$ for which the following series converges conditionally, converges absolutely, or diverges. What are the center, radius, and interval of convergence?

$$\sum_{n=0}^{\infty} \frac{(x+1)^{n+1}}{(n+2)2^{n-1}}.$$

**6**. (10 points) Compute (no shortcuts!!) the *Taylor series* centered at 0 for the function

$$f(x) = e^{1-\frac{3x}{4}}.$$

**7**. (10 points) Using what you know about familiar Taylor series, write a power series for $\arctan(2x)$. For what values of $x$ does your series converge?

(Hint: you should be familiar with the series for $\frac{1}{1-x}$.)

**8**. (10 points) Using what you know about familiar Taylor series, write a power series for $f(x) = \sin(2x)$. Similarly, write the the first four terms of a series for $g(x) = (\sin x)(\cos x)$. For what values of $x$ will these series converge? Use this to compute

$$\lim_{x \to 0} [f(x) - g(x)]$$

**9**. (10 points) Willy Wonka's newly patented *irresistable gobstopper* is highly addictive. There are currently 1000 gobstopper addicts. Every year, the number of addicts triples due to advertising, but only one tenth of these people survive schnozberry-extract withdrawal symptoms. Those who survive are drastically changed (so count as different people the next year). Write an infinite series which represents the total number of gobstopper addicts throughout the future. Will gobstopper addiction lead to extinction of the human race?

**10**. (10 points) Let $p_n$ denote the nth prime: $p_1 = 2$, $p_2 = 3$, $p_3 = 5$, $p_4 = 7$, $p_5 = 11$, etc. Determine whether the following series converges or diverges:

$$\sum_{n=1}^{\infty} \frac{1}{p_n^2}$$

**Bonus**. (10 points) State the *limit comparison test* for infinite series. Explain in words why the conclusions of this test make sense. Provide examples (1 each) of series for which this test is conclusive or inconclusive (as always, make sure to justify your claims).