Math 21C
Midterm 2
7/17/08

Name:

Signature:

Student ID:

- There are **ten** (plus cover and bonus) pages to the exam.
- The exam totals 100 points, plus 10 bonus points.
- You will have 50 minutes to complete the exam.
- No calculators, notes, or books allowed.
- Good luck!

Problem	Points	Your Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Bonus	10	
Total	100	

1. Definitions and Examples: let $\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$.

a. (2 points) Write the definition of $\mathbf{u} \cdot \mathbf{v}$. (an example is **not** sufficient for full credit)

b. (2 points) Write the definition of $\mathbf{u} \times \mathbf{v}$. (an example is **not** sufficient for full credit)

c. (2 points) Write the definition of $\| \mathbf{u} \|$. (an example is **not** sufficient for full credit)

d. (2 points) Write the definition of the projection $proj_{\mathbf{u}}\mathbf{v}$. (an example is **not** sufficient for full credit)

e. (2 points) Write the definition of the angle θ between $\mathbf{u}$ and $\mathbf{v}$. (an example is **not** sufficient for full credit)

2. Short Answers: let $\mathbf{u} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $\mathbf{u} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$

a. (5 points) Compute $\mathbf{u} + \mathbf{v}$, $\mathbf{u} - \mathbf{v}$, $\mathbf{u} \cdot \mathbf{v}$, and $2\mathbf{u}$.

b. (5 points) Sketch $\mathbf{u}$, $\mathbf{v}$, $\mathbf{u}+\mathbf{v}$, $\mathbf{u}-\mathbf{v}$, and $2\mathbf{u}$ in the coordinate plane. Make sure to label them!

3. (10 points) Compute the area of the triangle with vertices (1,0,2), (-1,3,0), and (0,0,5).

4. (10 points) Compute the distance between the point (0,1,-1) and the plane $x - y + 3z = 0$.

5. (10 points) Compute the volume of the parallelopiped with vertices (1,0,0), (1,2,3), (1,3,2), (1,0,1), (1,5,5), (1,2,4), (1,3,3), and (1,5,6).

6. (10 points) Write a parameterization for the line of intersection of the planes $2x - y = 0$ and $y - z = 1$.

7. (10 points) Write a parameterization for the line which is parallel to the plane $x + y - 2z = 1$, perpendicular to the line $\mathbf{p}(t) = \begin{pmatrix} 1-t \\ 1+t \\ 1 \end{pmatrix}$, and contains the point (0,0,1).

8. (10 points) Let $\mathbf{u} \neq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$, and $\mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$.

Prove or disprove:

If $\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{w}$ and $\mathbf{u} \times \mathbf{v} = \mathbf{u} \times \mathbf{w}$, then

$\mathbf{v} = \mathbf{w}$.

9. (10 points) Prove the parallelogram law: if $\mathbf{u}$ and $\mathbf{v}$ are vectors, then

$\| \mathbf{u} + \mathbf{v} \|^2 + \| \mathbf{u} - \mathbf{v} \|^2 = 2(\| \mathbf{u} \|^2 + \| \mathbf{v} \|^2)$

10. (10 points) let $\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$.

Prove or Disprove: $\mathbf{u} \times \mathbf{v} = -\mathbf{v} \times \mathbf{u}$

Bonus. (10 points) An example of a two-dimensional plane in four-dimensional space is the set of solutions to the equations

$$x + y + z + w = 1 \text{ and } x - y + z - w = 2.$$

Determine all four-dimensional vectors which are orthogonal to this plane. (Hint: start with two vectors parallel to the plane)