Why is there a node at the end of a pipe? in 2D

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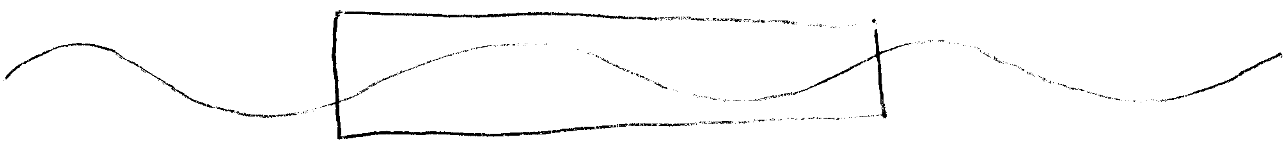
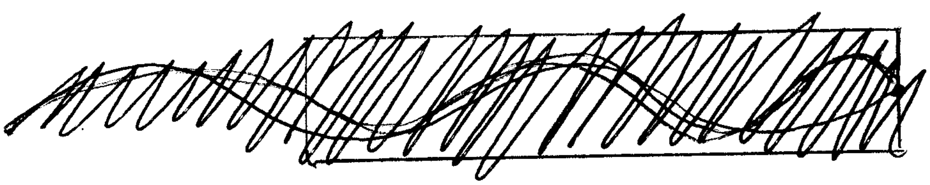
In studying soundwaves we come across a very interesting property that we can illustrate with open and closed organ pipes.

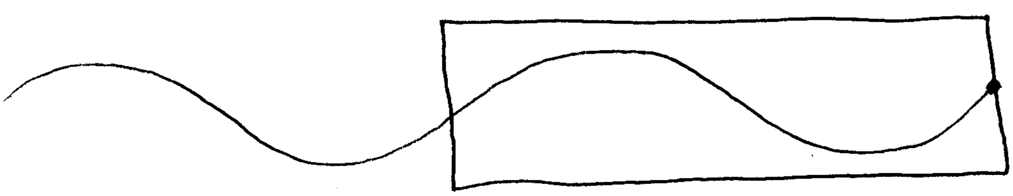
lets start by looking at a pipe that is open at both ends



The pipe has no affect on the sound wave.

what happens if we close one end?





We observe that there is a node at the closed end of the pipe

what happens if we close ~~to ends~~ both ends

We observe a node at both ends

What is happening?

There is two ways to go about explaining our observations.

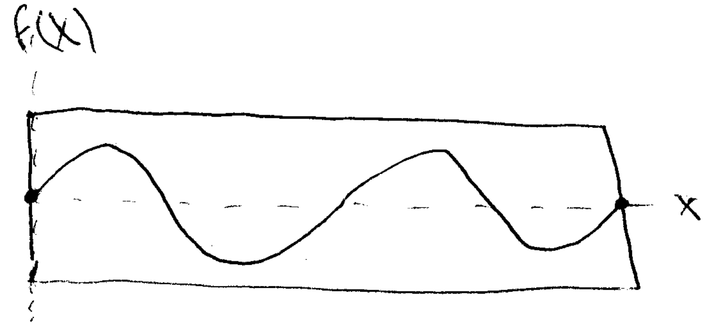
What is physically happening?

To physically explain this we have to go down to the Atomic level. Soundwaves need air or some type of medium in order to travel because the energy of the wave is passed from atom to atom. When the wave approaches the end of the pipe those atoms do not have enough energy to break the pipe so they are stuck inbetween the atoms next to them and the pipe. You can think of it in a quantum machanics sense will the pipe being an infinite potential well, It is not truely infinite, but since we cannot hear the atomic vibrations of the atoms that make up the pipe we can make the assumption that none of the soundwaves travelling through the pipe will have enough energy to tunnel through metal, which is why we find a node at the ends of the pipes.

The more Mathematical explanation

The mathematical explanation comes from Newton's 3rd law and applying boundary conditions.

lets start with the boundary conditions on a closed pipe



for ease of calculation we will place one node at the origin and the second at 2π

The wave is described by a linear combination of ~~sines and cos~~ sines and cosines

$f(x) = A\sin bx + B\cos bx$ apply boundary conditions

$f(0) = A\sin bx + B\cos bx \Rightarrow \sin(0) = 0$

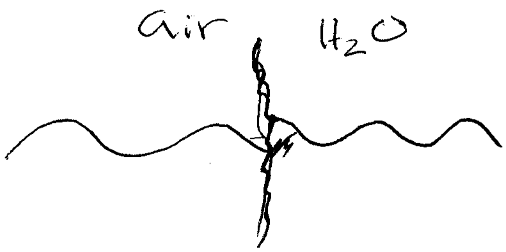
$\cos(0) = 1 \Rightarrow 1\cdot B = 0 \Rightarrow B = 0$

the wave is described by $A\sin bx$ which evaluated at 2π (b must be a integer) $= 0$ which explaines why we have a node at the end of the pipe

Now lets bring in more physics to explain what is going on.

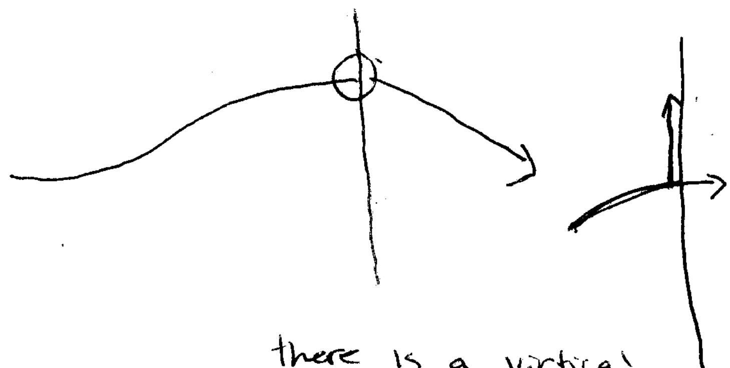
So this property of finding a node at the end of a pipe is a consequence of Newton's 3rd Law.

when waves travel from a less dense medium to a more dense medium it undergoes a phase flip



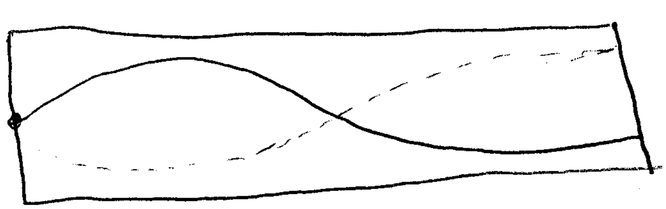
this doesn't happen if you go from more to less dense

the reason this happens is



there is a virtical component to the waves motion and to counter the upward force the wall exerts the equal and opposite force causing the phase flip.

So applying this to the closed pipe we observe



the wave ~~[illegible]~~ is distructively interfering with itself adding the two waves gives you a flat line across the pipe i.e. no sound

So the only way that we can hear sound out of a closed pipe is if there is a node at either end or else the wave destroys itself with destructive interference.

open pipes do not have this problem since the waves are just traveling from air to air, ~~and don't have~~ as a result there is no node at the open ends The relationship between open and closed pipes is quite simple. open pipes have the freedom of $\frac{1}{4}\lambda$ since there is no node but closed pipes have a node at both ends so they only have the freedom of $\frac{1}{2}\lambda$ so a open pipe twice as long as a closed pipe ~~could~~ would be equivalent to each other.