

Math 115A Homework 2

General Hint: you can try to use the Euclidean algorithm even when finding the gcd of two numbers that are expressed in terms of variables

1) Find the greatest common divisors below. Use any method you like but justify your answer.

- a) $(0, 113)$
- b) $(111, 129)$
- c) $(120, 165)$

2) Let $a \in \mathbb{Z}$ with $a > 0$. Find the greatest divisors below, justifying your answers.

- a) (a, a^n) where n is a positive integer
- b) $(a, a + 1)$
- c) $(a, a + 2)$
- d) $(3a + 5, 7a + 12)$

3) Find four integers that are relatively prime when taken together (so the greatest common divisor of all four is 1) but such that no two of the integers are relatively prime when taken separately. NOTE: the greatest common divisor of four numbers is just the largest integer that divides each of the four of them.

4) Let a and b be relatively prime integers. Prove that $(a + b, a - b)$ is either 1 or 2.

5) Let a and b be relatively prime integers. Find all possible values of $(a + 2b, 2a + b)$.

6) Let $a, b, c \in \mathbb{Z}$ with $(a, b) = 1$ and $c|(a + b)$. Prove that $(a, c) = 1$ and $(b, c) = 1$. *Hint: try a proof by contradiction for each claim.*

7) Use the Euclidean algorithm to find the gcd's below. Then express each $\gcd(a, b)$ as $aX + bY$ for some integers X and Y .

- a) $(793, 3172)$
- b) $(2059, 2581)$
- c) $(25174, 42722)$

8) Prove that one cannot express 7 as $18209X + 19043Y$ for some integers X and Y .

9) How difficult was this homework? How long did it take?