Math 115A Homework 3

1) Find the lest common multiples below:

a) $[24, 60]$

b) $[100, 105]$

c) $[101, 1111]$

2) Find all positive pairs of integers a, b such that $(a, b) = 12$ and $[a, b] = 360$.

3) a) Let $n \in \mathbb{Z}$ with $n > 1$. Prove that n is a perfect square if and only if all exponents in its prime factorization are even.

b) Let $n \in \mathbb{Z}$ with $n > 0$. Prove that n is the product of a perfect square and (possibly zero) distinct prime numbers.

4) A *powerful* integer $n > 1$ is one for which all exponents in its prime factorization are at least 2. Prove that a powerful number is the product of a perfect square and a perfect cube (so, it is of the form a^2b^3 for some $a, b \in \mathbb{Z}$).

5) Prove or disprove the following statements.

a) If $a, b \in \mathbb{Z}$, $a, b > 0$, and $a^2|b^3$, then $a|b$.

b) If $a, b \in \mathbb{Z}$, $a, b > 0$, and $a^2|b^2$, then $a|b$.

c) If $a \in \mathbb{Z}$, $a > 0$, and p is a prime such that $p^4|a^3$, then $p^2|a$.

6) a) Let $a, b \in \mathbb{Z}$. Prove that if a, b are both expressible as $6n + 1$ for some integer n, then ab is also expressible in that form.

b) Prove (without using Dirichlet's Theorem on Primes in Arithmetic Progressions) that there are infinitely many primes of the form $6n+5$ where $n \in \mathbb{Z}$. *Hint: try to mimic the proof that there are infinitely many primes of the form* $4n + 3$

7) How difficult was this homework? How long did it take?