Math 115A Homework 4

1) Find all positive integers m for which the following statements are true.

a) $13 \equiv 5 \pmod{m}$

b) $10 \equiv 9 \pmod{m}$

2) Let $a, b \in \mathbb{Z}$ such that $a \equiv b \pmod{m}$. If n is a positive integer such that $n|m$, prove that $a \equiv b \pmod{n}$.

3) Let $a, b \in \mathbb{Z}$ such that $a \equiv b \pmod{m}$. If c is a positive integer, prove that $ca \equiv cb \pmod{cm}$.

4) Let $a, b \in \mathbb{Z}$ such that $a \equiv b \pmod{m}$. Prove that $(a, m) = (b, m)$.

5) Let n be an odd integer not divisible by 3. Prove that $n^2 \equiv 1 \pmod{24}$.

6) Find all incongruent solutions to the following congruences.

a) $12x \equiv 16 \pmod{32}$

b) $623x \equiv 511 \pmod{679}$

c) $481x \equiv 627 \pmod{703}$

7) How difficult was this homework? How long did it take?