

## Math 115A Homework 7

- 1) a) Find the remainder of  $79^{79}$  divided by 9.  
b) Find the remainder of  $99^{99999}$  divided by 26.  
c) Find a multiplicative inverse of  $4^{67}$  modulo 19.
- 2) a) Let  $n$  be an integer not divisible by 3. Prove that  $n^7 \equiv n \pmod{63}$ .  
b) Let  $n$  be an integer divisible by 9. Prove that  $n^7 \equiv n \pmod{63}$ .
- 3) a) Prove that if  $n$  is an integer relatively prime to 72, then  $n^{12} \equiv 1 \pmod{72}$ .  
b) Find the largest integer  $m$  such that  $n^{12} \equiv 1 \pmod{m}$  for all integers  $n$  relatively prime to  $m$ . (*Note: by part (a) this number must be at least 72*).
- 4) Let  $m$  and  $n$  be relatively prime positive integers. Prove that  $m^{\phi(n)} + n^{\phi(m)} \equiv 1 \pmod{mn}$ .
- 5) Find  $\phi(n)$  for each value of  $n$  below.
  - a) 64
  - b) 105
  - c)  $15!$
- 6) Are there any positive integers  $n$  for which  $\phi(n) = n/4$ ? Why or why not?
- 7) Let  $m, n$  be positive integers such that  $m|n$ . Show that  $\phi(m)|\phi(n)$ . Is the converse true (i.e. is it true that if  $\phi(m)|\phi(n)$  then  $m|n$ )?
- 8) How difficult was this homework? How long did it take?