Math 115B Homework 2

1) Let $n \in \mathbb{Z}$ with $n > 0$ and let $\omega(n)$ denote the number of distinct prime numbers dividing n. Prove that

$$\sum_{d|n, d \geq 1} |\mu(d)| = 2^{\omega(n)}.$$

2) Let $n \in \mathbb{Z}$ with $n > 1$. If f is a multiplicative arithmetic function and $n = p_1^{a_1} p_2^{a_2} \cdots p_m^{a_m}$ is the prime factorization of n, prove that

$$\sum_{d|n, d \geq 1} \mu(d) f(d) = \prod_{i=1}^{m} (1 - f(p_i)).$$

3) Let $f(n) := 2n/3$ and g be an arithmetic function such that $f(n) = \sum_{d|n, d \geq 1} g(d)$. Determine the value of $g(12)$.

4) Let $n \in \mathbb{Z}$ with $n > 0$. Von Mangoldt's function, denoted $\Lambda(n)$, is defined to be

$$\Lambda(n) = \begin{cases} \ln p & \text{if } n = p^a \text{ for some } a \in \mathbb{Z} \text{ and } p \text{ prime} \\ 0 & \text{otherwise} \end{cases}$$

Prove that

$$\Lambda(n) = - \sum_{d|n, d \geq 1} \mu(d) \ln d.$$

5) Let f be an arithmetic function and, for $n \in \mathbb{Z}$ with $n > 0$, let

$$F(n) = \sum_{d|n, d \geq 1} f(d).$$

Prove that if F is multiplicative, then f is multiplicative. (Note that this is the converse of a theorem we proved in class)

6) How difficult was this homework? How long did it take?