Math 115B Homework 3

1) Find all incongruent solutions to $x^2 \equiv 1 \pmod 8$. Is it not true that all quadratic congruences have either no solutions or exactly two solutions? Explain.

2) Find all incongruent solutions of each quadratic congruence below.

a) $x^2 \equiv 23 \pmod{77}$ *(Hint: Consider the congruences* $x^2 \equiv 23 \pmod 7$ *and* $x^2 \equiv 23 \pmod{11}$ *and then apply the Chinese Remainder Theorem)*

b) $x^2 \equiv 11 \pmod{39}$

c) $x^2 \equiv 46 \pmod{105}$

3) a) Let p be an odd prime number. Prove that the $\frac{p-1}{2}$ quadratic residues modulo p are congruent to

$$1^2, 2^2, 3^2, \ldots, \left(\frac{p-1}{2}\right)^2$$

modulo p.

b) Let p be an odd prime number. Prove that the product of the quadratic residues modulo p is congruent to 1 modulo p if and only if $p \equiv 3 \pmod 4$. *(Hint: use Wilson's Theorem)*

4) Use Euler's criterion to evaluate the following Legendre symbols:

a) $\left(\frac{11}{23}\right)$

b) $\left(\frac{-6}{11}\right)$

c) $\left(\frac{5}{17}\right)$

5) Use Gauss's Lemma (Lemma 11.2) to evaluate the following Legendre symbols:

a) $\left(\frac{12}{23}\right)$

b) $\left(\frac{-5}{11}\right)$

6) In this problem, quadratic reciprocity may help. Evaluate the following Legendre symbols.

a) $\left(\frac{-79}{101}\right)$

b) $\left(\frac{87}{131}\right)$

c) $\left(\frac{91}{127}\right)$

a) $\left(\frac{-107}{211}\right)$

b) $\left(\frac{2817}{4177}\right)$ *(Note:* 2817 *is not prime)*

c) $\left(\frac{2819}{4177}\right)$ *(Note:* 2819 *is prime)*

7) (CHALLENGE: won't be graded)

a) Let p be a prime number with $p > 3$. Prove that the sum of the quadratic residues modulo p is divisible by p.

b) Let p be a prime with $p > 5$. Prove that the sum of the squares of quadratic residues modulo p is divisible by p.

8) How difficult was this homework? How long did it take?