

Math 115B Homework 4

1) a) Find the number of lattice points under the line $y = \frac{13x}{79}$ whose y coordinate is positive and whose x coordinate is ≤ 39 .

b) Use part (a) to evaluate the Legendre symbol $\left(\frac{13}{79}\right)$.

2) a) Describe completely all odd primes p modulo which 11 is a quadratic residue.

b) Describe completely all odd primes p modulo which 7 is a quadratic residue.

c) Describe completely all odd primes p modulo which 91 is a quadratic residue.

3) Let p and q be odd primes with $p = 4q + 1$. Prove that q is a quadratic residue modulo p .

4) Prove that if $\left(\frac{2}{n}\right)$ is the Jacobi symbol instead of the Legendre symbol, it is still true that it is 1 precisely when $n \equiv 1$ or $7 \pmod{8}$. Note: n has to be odd by definition. This problem will not be graded, so for your own learning's sake please try to come up with a solution yourself before checking your book.

5) a) Let p be an odd prime with $p \neq 2819$. Prove that

$$\left(\frac{2819}{p}\right) = (-1)^{(p-1)/2} \left(\frac{p}{2819}\right).$$

b) State and prove a similar result to the one in (a) for the Legendre symbol $\left(\frac{4177}{p}\right)$.

6) Let p and q be distinct prime numbers with $p \equiv q \equiv 3 \pmod{4}$. Prove that if the congruence $x^2 \equiv p \pmod{q}$ is not solvable, then the congruence $x^2 \equiv q \pmod{p}$ has exactly two incongruent solutions modulo p .

7) a) 11.3.1 in your book

b) Prove that if a is a quadratic residue modulo n , then $\left(\frac{a}{n}\right) = 1$, but not conversely.

8) How difficult was this homework? How long did it take?