Math 115B Homework 6

1) Let x, y, z be a primitive Pythagorean triple with y even.

a) Prove that exactly one of x and y is divisible by 3.

b) Prove that exactly one of x and y is divisible by 4.

c) Prove that exactly one of x, y, and z is divisible by 5.

d) Prove that $x + y \equiv x - y \equiv 1$ or $7 \pmod 8$.

e) Prove that $60|xyz$.

2) Prove that every positive integer greater than 2 is part of at least one Pythagorean triple. *Hint: let $n \in \mathbb{Z}$ with $n > 2$. If n is odd, consider the triple n, $(n^2 - 1)/2$, $(n^2 + 1)/2$. If n is even, consider the triple n, $(n^2/4) - 1$, $(n^2/4) + 1$.*

3) a) Prove that there are infinitely many primitive Pythagorean triples x, y, z with $y = x + 1$. *Hint: if x, y, z is such a triple, consider the triple* $3x + 2z + 1, 3x + 2z + 2, 4x + 3z + 2$.

b) Prove that every primitive Pythagorean triple x, y, z with $y = x + 1$ can be generated using the procedure motivated in part (a).

4) Find all solutions in positive integers of each Diophantine equation below.

a) $x^2 + 2y^2 = z^2$ *Hint: parallel the general technique used in the proof of parametrization of primitive Pythagorean triples (Theorem 13.1).*

b) $x^2 + 3y^2 = z^2$

c) $x^2 + 4y^2 = z^2$

d) $x^2 + py^2 = z^2$ where p is a prime number

5) How difficult was this homework? How long did it take?