Math 115B Homework 7

1) Determine whether each integer is expressible as the sum of two squares of integers.

   a) 103

   b) 207

   c) 637

   d) 6!

   e) 10!

   f) 1989^{1989}

2) Determine whether each integer is expressible as the sum of three squares of integers.

   a) 324

   b) 496

   c) 3008

   d) 28672

   e) 10!

   f) A prime number p with $\left(\frac{2}{p}\right) = -1$.

3) Find all integers between 100 and 200 that are not expressible as the sum of three squares of integers.

4) Prove that a positive integer is expressible as the difference of two squares of integers if and only if it is not of the form $4n + 2$ for some integer n.

5) Prove that any odd positive integer is expressible as the sum of four squares of integers, two of which are consecutive. *Hint: show that any positive integer of the form $4n+1$ is expressible as $(2a + 1)^2 + (2b)^2 + (2c^2)$ for some integers a, b, c. Then show that $2n + 1$ is expressible as the sum of four squares of integers, two of which are a and $a + 1$.*

6) Prove that every integer is expressible as the sum of five cubes of integers. *Hint: the identity $n^3 - 6a = n^3 + a^3 + a^3 - (a + 1)^3 - (a - 1)^3$ may be useful.*

7) Problems 13.4.2 and 13.4.10 in your book.

8) How difficult was this homework? How long did it take?